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Module 3



# Applied Mathematics 20

## LINEAR SYSTEMS

Learning Technologies Branch

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Module
3



# Applied Mathematics 20

## LINEAR SYSTEMS

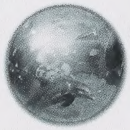
Learning Technologies Branch

Alberta LEARNING

Applied Mathematics 20
Module 3: Linear Systems
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1980-2

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| This document is intended for | |
|---|---|
| Students | ✓ |
| Teachers | ✓ |
| Administrators | |
| Home Instructors | |
| General Public | |
| Other | |



You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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# Welcome

***Applied Mathematics 20***

Module 1
Graphs

Module 2
Non-Linear Functions

Module 3
Linear Systems

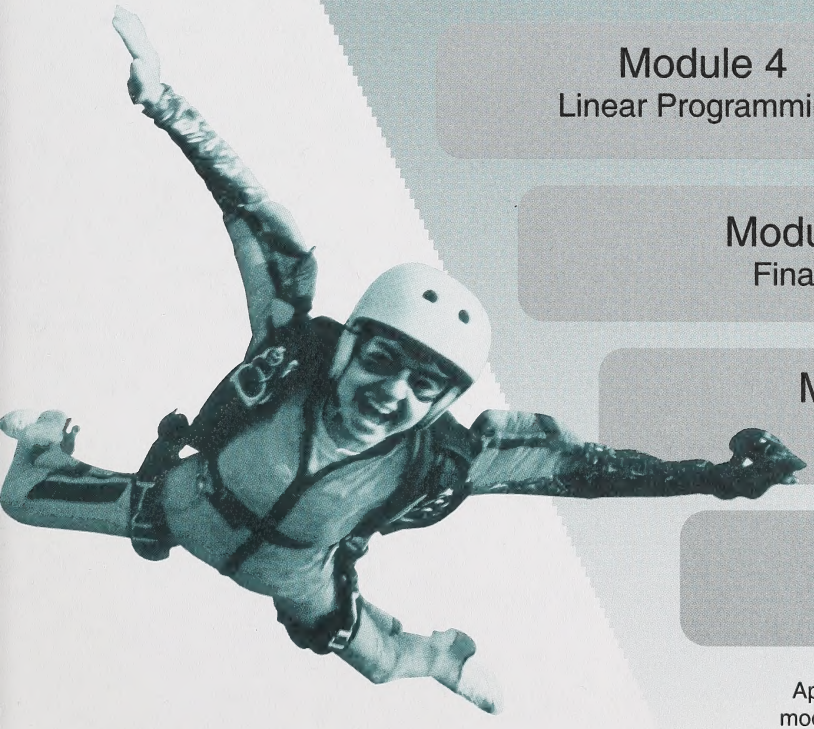
Module 4
Linear Programming

Module 5
Finance

Module 6
Circles

Module 7
Measurement

*Welcome to Module 3. We hope you'll enjoy your study of Linear Systems.*



Applied Mathematics 20 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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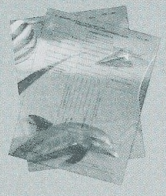
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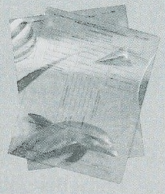
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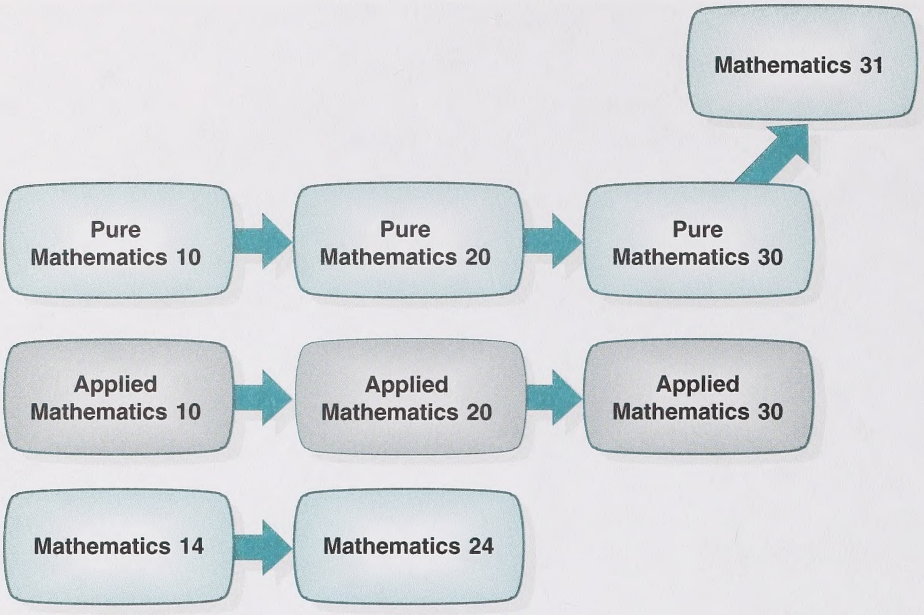


*Submit the Module Assignment.*

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# Introduction to Applied Mathematics 20

Applied Mathematics 20 is the second course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.




Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read "Questions and Answers About Senior High School Mathematics" and "The New Senior High School Mathematics Program and Post-Secondary Studies." These documents can be found at the following Internet site:

**http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math**

Before enrolling in Applied Mathematics 20, it is recommended that you talk with a school counsellor about your career plans.



## TRANSFERRING FROM THE APPLIED PROGRAM

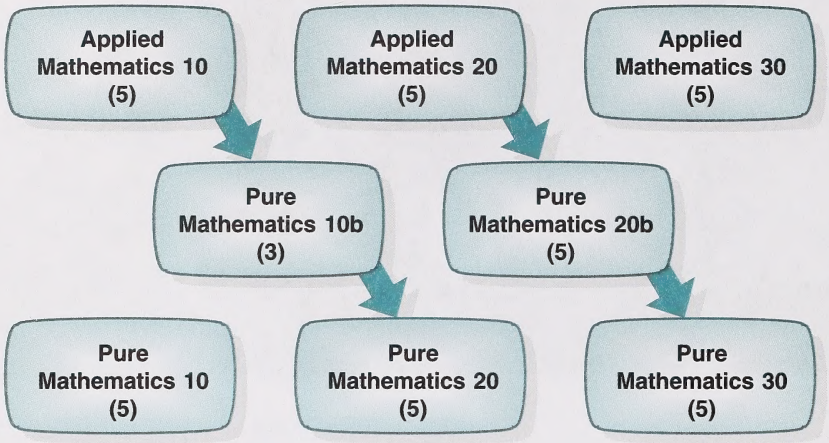
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

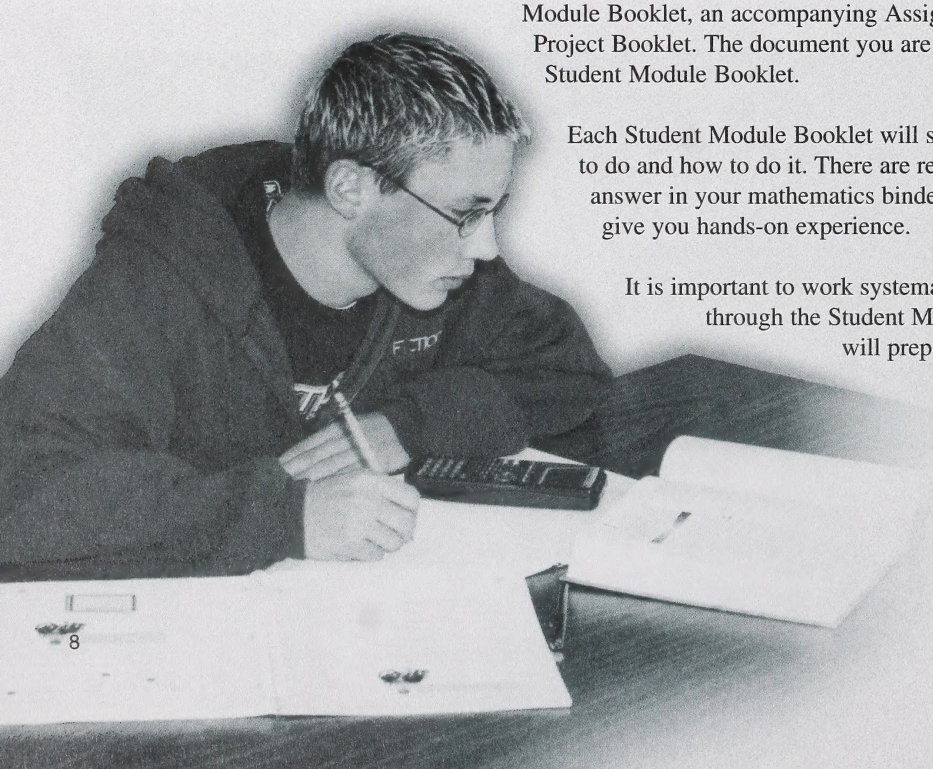
| Applied Topics | Common Topics | Pure Topics |
|---|---|---|
| • linear programming<br>• data tables and trends<br>• design and layout<br>• metric and imperial measure<br>• data presentation<br>• vectors and matrices<br>• periodic, fractal, and recursive patterns<br>• financial decision making<br>• costing and design problems | • spreadsheets<br>• line segments and linear graphs<br>• scaling<br>• triangles<br>• surveys<br>• financial mathematics<br>• quadratic functions<br>• circle geometry<br>• the bell curve | • irrational numbers<br>• exponents<br>• polynomial and rational expressions<br>• mathematical expectation<br>• growth patterns<br>• linear and non-linear systems<br>• operations on functions<br>• mathematical reasoning<br>• exponential and logarithmic functions<br>• conics<br>• combinations<br>• trigonometric functions |

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.




## STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 20



For each module in Applied Mathematics 20, there is a Student Module Booklet, an accompanying Assignment Booklet, and a Project Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step-by-step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words.

  Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

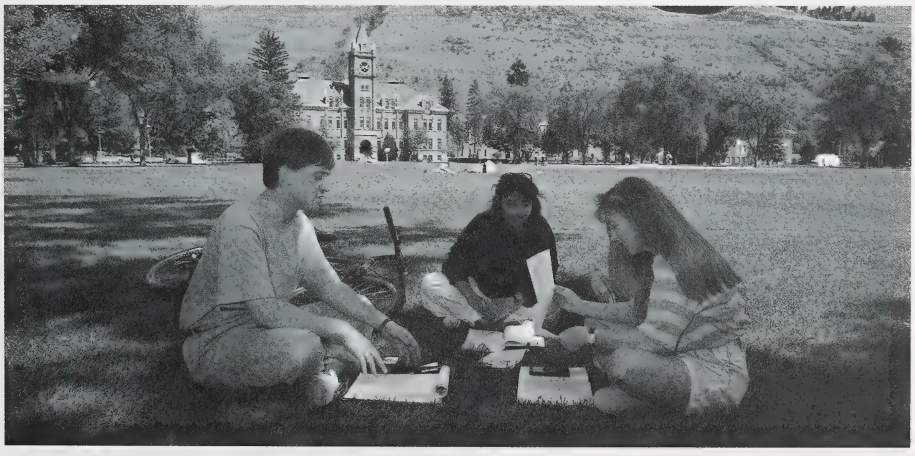
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

## Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.



## Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 20, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 11 Source Book* Western Canadian Edition, published by Addison Wesley Longman Ltd. (2000)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  **Note:** Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.

- a graphing calculator

  **Note:** Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

| Texas Instruments | Sharp | Casio | Hewlett-Packard |
|---|---|---|---|
| TI-82**<br>TI-83<br>TI-83 Plus<br>TI-86<br>TI-89<br>TI-92*<br>TI-92 Plus | EL-9600C<br>EL-9600*<br>EL-9200*<br>EL-9300* | Algebra FX 2.0<br>CFX-9850 GA-Plus*<br>CFX-9850 G*<br>CFX-9800 G*<br>FX-9700 series* | HP 39g |

*no longer commercially available

**The TI-82 calculator will remain on the approved list for the 2000–2001 and 2001–2002 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources that you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

**http://www.lrc.learning.gov.ab.ca**

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

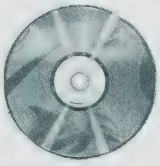
## Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

- Refer to the textbook.



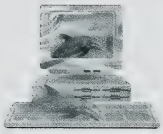
- Refer to the Applied Mathematics 20 CD.



- Use mathematical instruments or measuring tools.



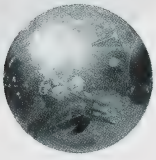
- Work with a computer.



- Complete the module project or assignment.



- Explore the Internet.



**Remember:** Any Internet website address given in this module is subject to change.

# MODULE OVERVIEW

## Four-year-old skydiver leaps for record books

November 8, 2000
CAPE TOWN, South Africa (Reuters)—The parents of Toni Stadler hope he has leaped into the record books by becoming the youngest person to skydive, at the age of four. …

Toni's tandem parachute jump took place at the Cape Parachute Club, 25 miles north of Cape Town, on Oct. 27, five days before his fifth birthday. …

"We have been skydiving for four years and recently Toni started asking if he could jump too," Daniela said. "We did not make him do it. He made us let him."

The youngster was strapped to jump master Paul Lutge's chest as they leaped out of their single-engined plane 10,000 feet above the earth, freefalling for half a minute before opening the parachute.

"I would like to do it again, but next time without the somersault as we jump out of the plane. It was fine without the somersault," Toni said …

## Ex-President Bush makes birthday parachute jump

June 9, 1999
COLLEGE STATION, Texas
(AllPolitics, June 9)—Proving that "old guys can still do stuff," former President George Bush kicked off his upcoming 75th birthday with a 12,500 foot parachute jump Wednesday.

The jump—his third—landed Bush on the lawn in front of his presidential library at Texas A&M University in College Station, Texas, about 90 miles northwest of Houston.

Bush, whose birthday is Saturday, described his free fall as "a euphoric high," adding "to end up landing safely … you just feel wonderful."

"We were rolling around in the sky, unlike the first jump and it was heaven. It was just exhilarating," he said. …

[1] "Four-Year-Old Skydiver Leaps for Record Books," *CNN.com*, 8 November 2000, <http://www.cnn.com/2000/WORLD/africa/11/08/safrica.skydive.reut/> (2 February 2001). Reprinted by permission of Reuters.
[2] "Ex-President Bush Makes Birthday Parachute Jump," *CNN.com*, 9 June 1999, <http://www.cnn.com/ALLPOLITICS/stories/1999/06/09/bush.skydive> (2 February 2001). Reprinted by permission Cable News Network LP, LLLP.

As long ago as the 1100s, the Chinese jumped from hills and cliffs using parachute-like structures. In the 1800s, people in Europe used parachutes to jump from balloons. By World War II, 1939–1945, parachutes were used to a wide extent in military operations.

Skydiving involves jumping from an airplane and parachuting to the ground for sport. Improvements in the design of parachutes have made parachutes very safe and maneuverable. With these improvements, skydiving has grown in popularity.

When skydiving—after jumping from the airplane—the pull of gravity causes you to fall through the air faster and faster. Eventually, the air resistance is enough to keep the speed of your free fall from increasing and you drop at a very fast, but stable, speed. Once you pull the ripcord to open your parachute, the parachute comes out of its container and is inflated by the air rushing past. Once inflated, the parachute provides enough air resistance to limit the speed of your descent for a safe landing.



Linear systems of equations—two or more linear equations working together—can be used to model the fall of skydivers through the air before and after their parachutes open.

In this module, you will solve linear systems of equations graphically and algebraically. You will discover distinguishing properties of linear systems and use linear systems to model problems and solve problems.

## ASSESSMENT

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

| | |
|---|---|
| **Module Project** | **40 marks** |
| **Module Assignment** | **60 marks** |
| **TOTAL** | **100 marks** |

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

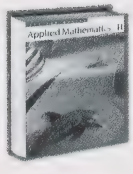
# MODULE PROJECT
## SKYDIVING



## Beginning the Project

Would you like to parachute out of an airplane high above the ground? Many people do just that—simply for the sheer excitement.

Your module project for Module 3: Linear Systems is Skydiving. You will use graphs and linear equations to model the motion of a skydiver. You will research skydiving to enable you to apply your graphs and linear equations in a meaningful way. Your project will be marked on the completeness and accuracy of the information and the mathematical representation of the descent of a skydiver in a dive position and in a spread stable position.



To begin your project, turn to page 112 of the textbook and read "Skydiving," answering the questions posed near the bottom of the page. Store your answers to these questions in the project section of your mathematics binder.

After you have recorded some of your initial ideas regarding the project, begin researching some of the things you may need to know about parachutes and skydiving.

To start, you may find it useful to visit Addison-Wesley's Internet site, described on page 113 of the textbook. You may also go to the following encyclopedia sites to do research for the project:

**http://encarta.msn.com**
**http://www.britannica.com**

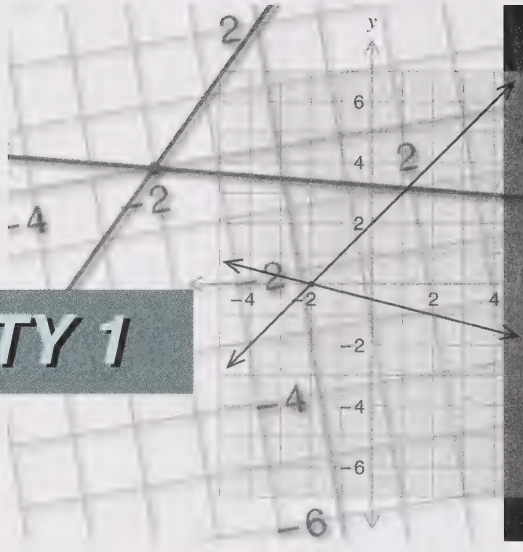
As you work through Activities 1 to 5 in this module, continue your research for this project. You may do your research by visiting skydiving companies and talking to skydiving instructors and by surfing the Internet, reading magazines, and visiting the local library.

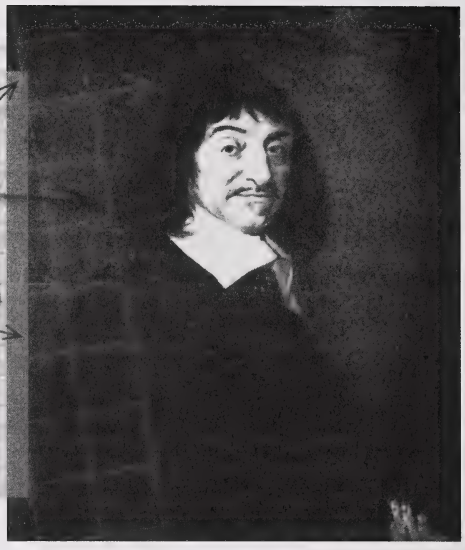




In this project you will focus on the linear relations between vertical distance dropped and the elapsed time of descent. To analyse these relations, you will need a good understanding of linear systems. As you work through Activities 1 to 5, you will develop the required understanding of linear systems.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to talk about this project with your study partner or with a family member. Remember, the work on the project that you submit must be your own.



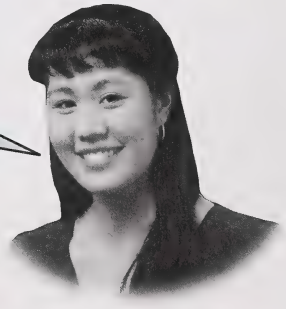
# ACTIVITY 1

## Solving Linear Systems by Graphing

It was in the writing of René Descartes, a French philosopher of the seventeenth century, where the idea of using a coordinate plane to represent equations was introduced. When you graph an equation, you create a picture that allows you to visualize the properties of the equation. On a coordinate plane, a linear equation appears as a line. The ordered pair for any point on the line results in the left side of the equation equaling the right side.

Now, suppose you have a system of two linear equations. Here, any ordered pair that satisfies both equations of the system is said to be a solution of the **linear system**.

In this activity, you will explore how the graph of a linear system can be used to determine the solution of that system.



Without looking at the graph, you know that a point satisfies an equation when its coordinates make the left side and right side of the equation equal.

## Example

Determine whether the point $(-9, 2)$ satisfies the equation $3y = x + 15$.

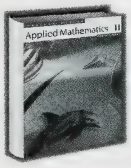
### Solution

| LS | RS |
|---|---|
| $3y$ | $x+15$ |
| $=3(2)$ | $=-9+15$ |
| $=6$ | $=6$ |

LS = RS

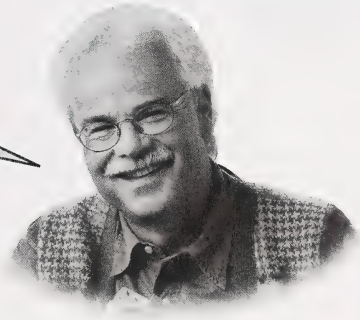
The point $(-9, 2)$ satisfies the equation.

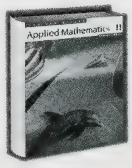
By checking whether a point satisfies **both** equations of a system, you can determine whether a point is a solution of the system.




**1.** Turn to page 121 of the textbook and answer exercise 1 of "Exercises: Checking Your Skills."

**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 62–63.**



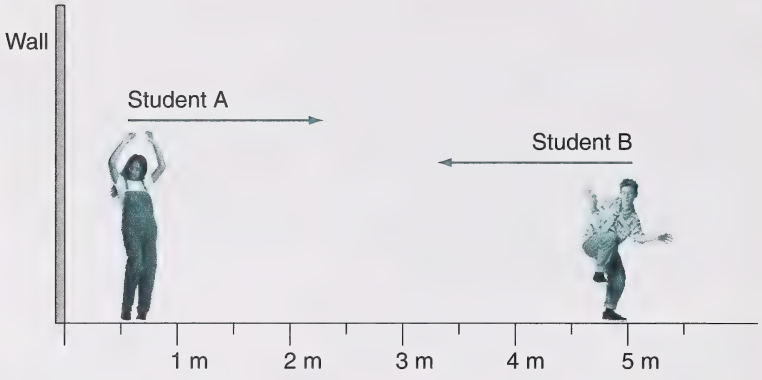

To explore how graphs can be used, you will need to brush up on your graphing skills. Turn to page 433 of the textbook and read the entry "linear equation and its graph." Pay special attention to the two methods of graphing an equation: the slope-intercept method and the intercept method.

**2.** Answer exercises 1 to 5 of "Practice Your Prior Skills" on page 114 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 64–65.**

In the next investigation you will use graphing to analyse motion. The analysis will help you see how graphing can be used to find a point that satisfies a linear system, that is to find a solution of the system.

In the investigation you will mark off a distance of 5 m on masking tape (as shown in the diagram). You will have two people shuffling in opposite directions.

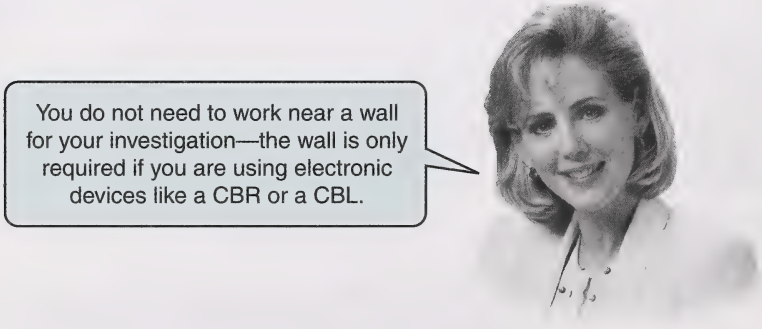


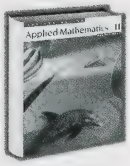

Student A will start at the 0.5-m position on one side of the tape and shuffle away from the wall; student B will start at the 5-m position and shuffle in the opposite direction, towards the wall, on the other side of the tape.



Turn to pages 115 to 117 of the textbook and read "Investigation 1: Meeting and Greeting."

If you have access to a CBL or a CBR, complete exercises 1 and 2. If you are not using technology, complete exercises 1 and 3. Try to work in a group of four. It's best to have two people shuffling toward each other at the same time and two people timing with a stopwatch. However, if you have only one other person helping and only one stopwatch, you can be Student A and your helper Student B. You and then your helper can shuffle the whole distance while the other does the timing.






**3.** Turn to pages 116 and 117 of the textbook and answer exercises 4 to 8 of "Investigation 1: Meeting and Greeting." **Note:** If you did not have a straight path of 5 m to do the shuffling or lacked some other requirements of the investigation, use the following data.

| | Start | | Finish | |
|---|---|---|---|---|
| | Time (s) | Distance from Wall (m) | Time (s) | Distance from Wall (m) |
| Student A | 0.0 | 0.5 | 5.0 | 5.0 |
| Student B | 0.0 | 5.0 | 5.0 | 2.5 |

**4.** Turn to page 120 of the textbook and answer exercises 1 and 2 of "Discussing the Ideas."

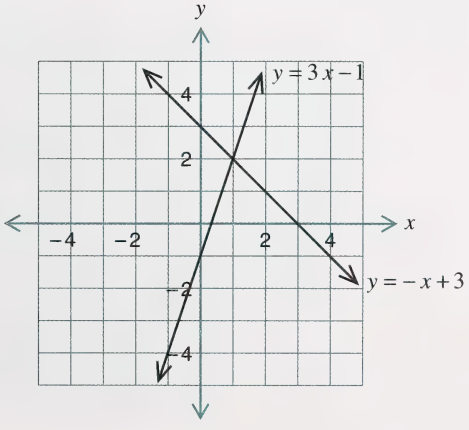
**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 66–68.**

## SOLVING SYSTEMS GRAPHICALLY USING PENCIL AND PAPER

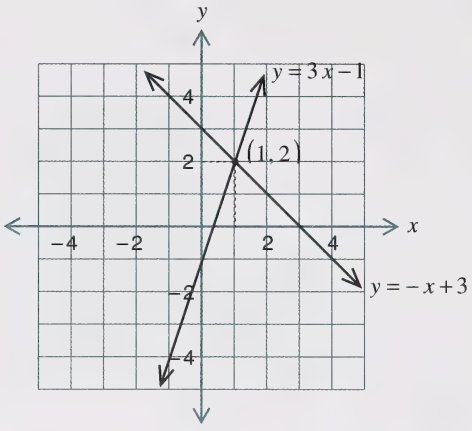
In the preceding investigation, you discovered that the intersection point of two graphs had a special significance.

What can you say about the intersection point of the graphs of a system of equations? The coordinates of the point of intersection form a solution of the system. For example, here is a system of equations and its graphs.

$y = 3x - 1$ (1)
$y = -x + 3$ (2)



The intersection point shows that the ordered pair $(1, 2)$ is a solution of the system.






Turn to page 117 of the textbook and read the text in the shaded rectangle.

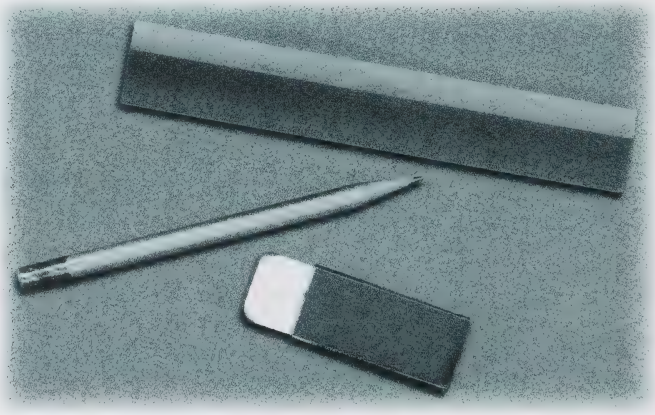
In the previous investigation, the solution corresponded to when and where the shufflers arrived at the same position with respect to the wall. Note that system solutions may take on other meanings. For example, the solution of certain systems of equations in business corresponds to a **break-even point**, where there is equal cost and revenue for a business.

Turn to pages 117 and 118 of the textbook and work through "Example 1: Determine the break-even point of a business."

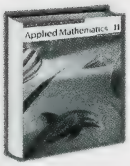
**5.** Answer exercise 4 of "Discussing the Ideas" on page 120 of the textbook.

**6.** Answer exercises 2.a., 2.c., 3.a., and 7 of "Exercises: Checking Your Skills" on pages 121 and 122 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 68–71.**



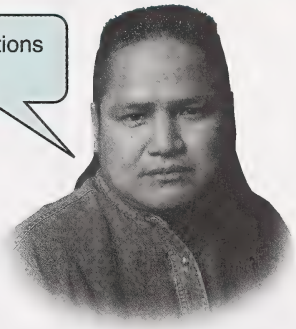
## SOLVING SYSTEMS GRAPHICALLY USING TECHNOLOGY



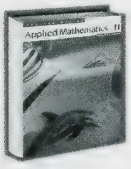

Turn to page 118 of the textbook and read the paragraph preceding Example 2; then work through "Example 2: Determine the solution of a linear system graphically using technology" on pages 118 to 120.

**7.** Answer exercises 6, 7, and 8 of "Discussing the Ideas" on page 120 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 71–72.**




## Example




Turn to page 121 of the textbook and answer exercise 4 of "Exercises: Checking Your Skills."

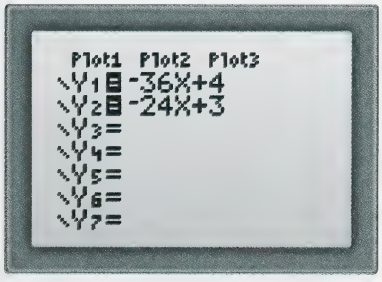
### Solution

**4. a.** Let $a$ represent the independent variable and $b$ represent the dependent variable.

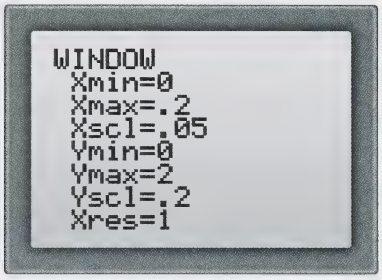
**Step 1:** Rearrange the equations so $b$ is a function of $a$.

$$4 = b + 36a \qquad 3 = b + 24a$$
$$-36a + 4 = b \qquad -24a + 3 = b$$
$$b = -36a + 4 \qquad b = -24a + 3$$

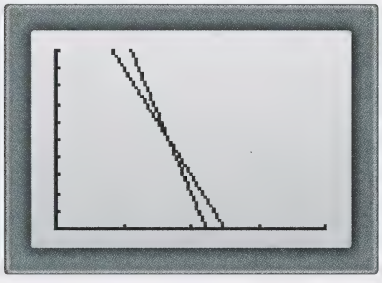
**Step 2:** Enter the system into the equation editor of your graphing calculator.



**Step 3:** Choose appropriate window settings to show the point of intersection of the system clearly. **Note:** It is not uncommon to use trial and error until you get a suitable display.

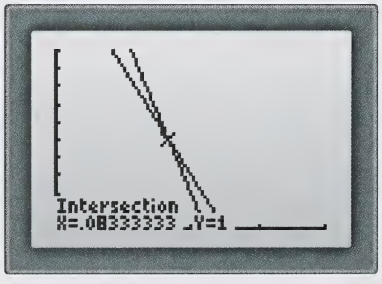



**Step 4:** Graph the system of equations.



**b. Step 1:** Use the Intersect feature from the CALCULATE menu to find the intersection point.

2nd [ CALC ] 5 (5:intersect) ENTER ENTER ENTER



**Step 2:** Identify the values that satisfy the system.

The ordered pair $(0.08\overline{3}, 1)$ satisfies the system. Therefore, $a = 0.08\overline{3}$ and $b = 1$.

**Step 3:** Check that the values satisfy the system.

| LS | RS |
|---|---|
| 4 | $b + 36a$ |
| | $\doteq 1 + 36(0.08333)$ |
| | $\doteq 1 + 2.99988$ |
| | $\doteq 4$ |
| LS | = RS |

| LS | RS |
|---|---|
| 3 | $b + 24a$ |
| | $\doteq 1 + 24(0.08333)$ |
| | $\doteq 1 + 1.99992$ |
| | $\doteq 3$ |
| LS | = RS |

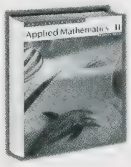
Therefore, the mass of the crate is 1 kg, and the mass of each apple is approximately 0.083 kg.



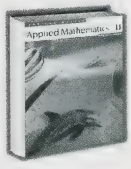
For additional instruction on using the TI-83 graphing calculator to solve systems of linear equations, view the segment titled *Solving Systems of Linear Equations Using the TI-83* on the Applied Mathematics 20 CD.



**8.** Turn to page 121 of the textbook and answer exercises 5 and 6 of "Exercises: Checking Your Skills."




**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 72–73.**




Turn to page 122 of the textbook and look ahead to exercise 8 of "Exercises: Checking Your Skills." In exercise 8.c., you have to determine the difference in the cost ($y$-values of the equations) when the number of hours ($x$-values) takes on a certain value.

The next example will show you how to determine the difference in the $y$-values between the two lines of a simple system by using the table of values and by using the Value feature from the CALCULATE menu. Carefully follow each method to see which one is more effective.

## Example

Use this system of equations to answer the questions that follow:

$$y = x + 2 \quad (1)$$
$$y = 2x \quad (2)$$

**a.** Using the table of values, determine the difference in the $y$-values when $x = 1.4$.

**b.** Using the table of values, determine the difference in the $y$-values when $x = 4.5$.

**c.** Using the Value feature from the CALCULATE menu, determine the difference in the $y$-values when $x = 4.5$.

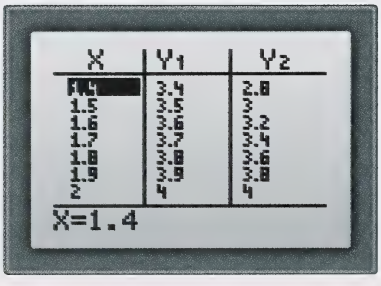
### Solution

**a.** **Step 1:** Set the table of values so it shows the $y$-values for $x = 1.4$.

Press 2nd [ TBLSET ], and set up the table as follows.




**Step 2:** Press 2nd [ TABLE ] to display the table of values.



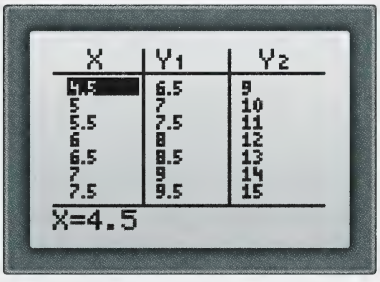

The difference in $y$-values is $3.4 - 2.8 = 0.6$.

**b.** **Step 1:** Set the table of values so it shows the $y$-values for $x = 4.5$.

Press [2nd] [ TBLSET ], and set up the table as follows.



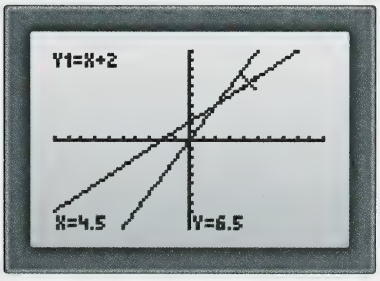

**Step 2:** Press [2nd] [ TABLE ] to display the table of values.




The difference in the $y$-values is $9 - 6.5 = 2.5$.

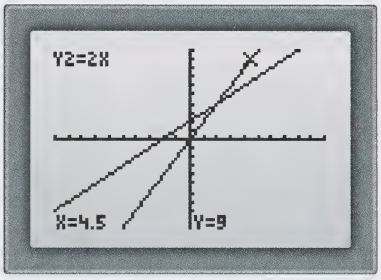
**c.** **Step 1:** Use the Value feature from the CALCULATE menu to determine the value of $y$ when $x = 4.5$.

[2nd] [ CALC ] [1] (1:value) [4] [•] [5] [ENTER]

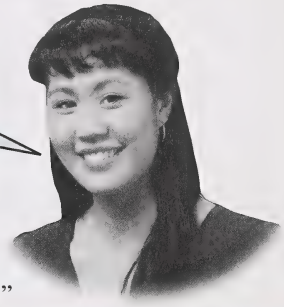



Use the standard window settings, [ZOOM] [6].

**Step 2:** To determine the value of $y$ in $y = 2x$ when $x = 4.5$, press ▲. **Note:** You could use ▼ to switch graphs as well.




The difference in the $y$-values is $9 - 6.5 = 2.5$.



The Value feature is very effective! You may use either of these methods to determine the difference in $y$-values.



**9.** Turn to pages 122 to 124 of the textbook and answer exercises 8 and 11 of "Exercises: Checking Your Skills."

**Compare your responses with the suggested answers in the Appendix, Activity 1, pages 74–79.**

## LOOKING BACK

In this activity you solved linear systems of equations by graphing both manually (using pencil and paper) and using a graphing calculator. You then applied your knowledge of solving systems of linear equations to solving real-world problems.



**10.** Turn to page 124 of the textbook and answer "Communicating the Ideas."

**Compare your response with the suggested answer in the Appendix, Activity 1, page 79.**

# ACTIVITY 2

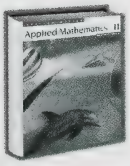


## Algebraic Modelling

The manuscripts of the Ancient Egyptians show that symbols were used to represent numbers as early as 1700 B.C. These symbols were used to help discover the relationships between numbers.

Letters of the alphabet are the symbols used to represent numbers in modern algebra. A letter used this way is called a **variable**. One or more variables make up algebraic equations. Algebraic equations that represent the relationships of a real-world situation are created using algebraic modelling.

In this activity you will develop the skill of algebraically modelling problem situations with linear systems of equations.




Turn to page 125 of the textbook and read the introductory paragraphs of Tutorial 3.2, "Algebraic Modelling." Then work through "Example 1: Reword statements" on pages 125 and 126.

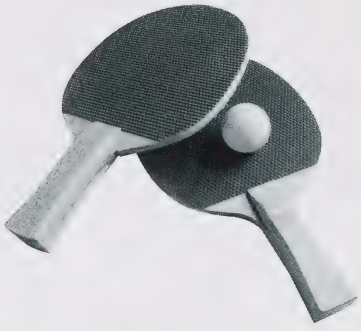
1. Answer exercises 1 and 2 of "Exercises: Checking Your Skills" on page 129 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 2, page 80.**

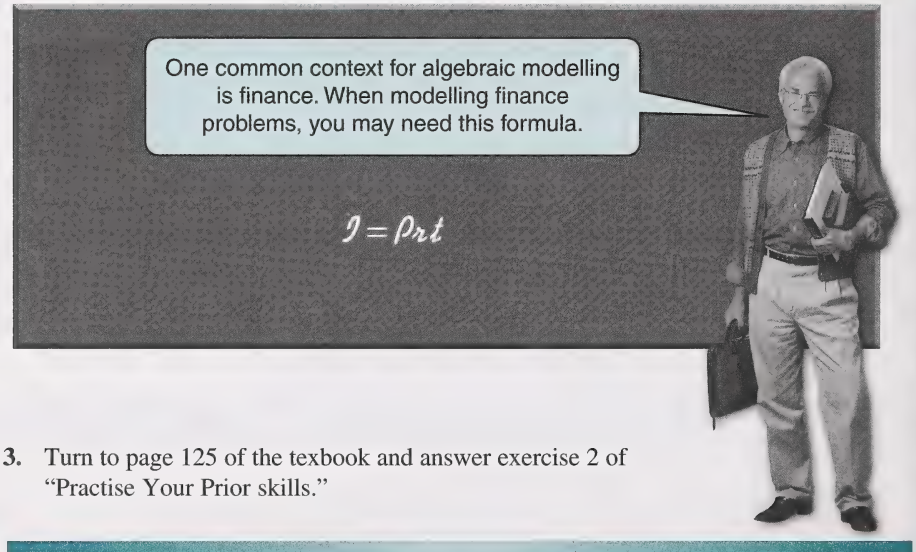


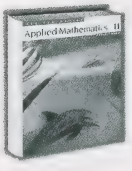
Using a diagram often helps to process information given in a problem. Turn to pages 126 and 127 of the textbook and work through "Example 2: Draw a diagram to illustrate a problem."

**2.** The perimeter of a regulation table for table tennis is 853.0 cm. The difference between the length and width of the table is 121.5 cm. Write equations showing the relationship between the length and the width of the table.



**Compare your response with the suggested answer in the Appendix, Activity 2, page 81.**



**3.** Turn to page 125 of the texbook and answer exercise 2 of "Practise Your Prior skills."

**Compare your response with the suggested answer in the Appendix, Activity 2, page 81.**

The next example, from the world of finance, shows how a data table can be used to help set up the equations required to determine the amount invested at each rate. A table will show relationships clearly.



Turn to page 127 of the textbook and work through "Example 3: Use a table to organize information." **Note:** The wording in the first column, third row should read Second Investment, not First Investment.

**4.** Answer exercises 6 and 11 of "Exercises: Checking Your Skills" on pages 129 and 130 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 2, page 82.**

From the science courses you have taken so far, you may have come across the following formula:

$$d = vt \quad \begin{cases} d = \text{distance} \\ v = \text{velocity (speed)} \\ t = \text{elapsed time} \end{cases}$$

This formula shows how distance, velocity, and time are related. Distance is equal to speed multiplied by elapsed time. Many interesting problems arise from the study of objects in motion.



**5.** Turn to page 125 of the textbook and answer exercise 1 of "Practise Your Prior Skills."

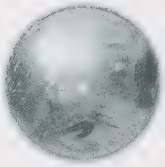
**Compare your response with the suggested answer in the Appendix, Activity 2, page 83.**

Have you ever been on a cross-Canada or transcontinental flight and arrived early? If so, you may have heard an announcement from the pilot somewhat like this.



When in an airplane, the air speed affects how fast the plane is "actually" flying. Likewise, when in a boat, the current affects the actual speed of the boat.

Using diagrams and data tables may help you model situations relating to motion in moving air or water.



You may wish to visit the following Internet site for an interesting exercise involving real-time flights and determining average speed over the duration of a flight.

**http://www.ramsey.k12.nj.us/webquest/projects/rhs/science/velocity/index.htm**





Turn to page 128 of the textbook and work through "Example 4: Draw a diagram and use a table to interpret a problem."

**6.** Answer exercises 8 and 10 of "Exercises: Checking Your Skills" on page 130 of the textbook. **Note:** In exercises 8 and 10, it is not intended that you calculate the distance travelled.

**Compare your responses with the suggested answers in the Appendix, Activity 2, pages 83–84.**

## LOOKING BACK

In this activity, you learned to model problem situations algebraically.



**7.** Turn to page 129 of the textbook and answer exercise 1 of "Discussing the Ideas."

**8.** Turn to page 130 of the textbook and answer "Communicating the Ideas." Choose exercise 5 on page 129 for your illustration of how either one or two unknown quantities (e.g., variables) can be used.

**9.** Turn to page 129 of the textbook and answer exercise 3 of "Discussing the Ideas."

**Compare your responses with the suggested answers in the Appendix, Activity 2, pages 84–86.**

# ACTIVITY 3



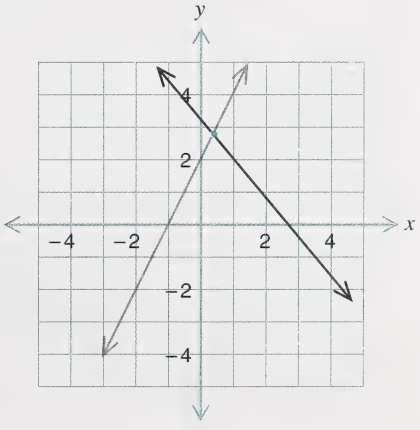

LOUIS ST. LAURENT HIGH SCHOOL

## Solving Linear Systems by Substitution

In a basketball or volleyball game, when a coach pulls a player out of a game and replaces him or her with another, a substitution is being made.

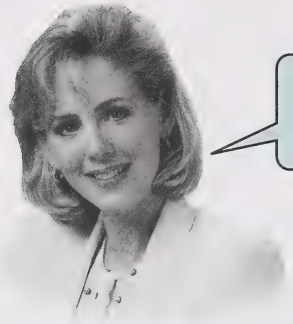
Similarly, in algebra, when you replace a variable with other variables or numbers, a substitution is being made. In this activity you will use substitution to solve linear systems.

Why introduce another method of solving systems of equations? After all, graphing allows you to visualize the equations of a system and the solution to the system. However, graphing may not accurately indicate the solution to a system. For example, if the ordered pair $\left(\frac{2}{5}, 2\frac{3}{4}\right)$ satisfies the equations of a system (like the one shown in the graph on the right), the $x$- and $y$-values may not be easily determined from the point of intersection of the lines.



Algebraic methods of solving systems of equations, on the other hand, can give you precise values. Furthermore, algebraic methods can be more convenient.

Substitution is one algebraic method of solving a system of equations.



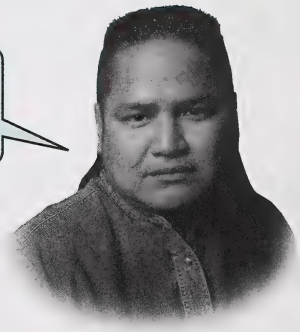
When using algebraic methods, you must remember how to deal with brackets and fractions in equations.



**1.** Turn to page 131 of the textbook and answer exercises 1.a., 1.c., and 2.c. of "Practise Your Prior Skills."

**Compare your responses with the suggested answers in the Appendix, Activity 3, page 86.**

You must also recall how to algebraically solve a single equation in which only one variable appears.



You will be converting a system of equations into a single equation having only one variable. That single equation is the one you will have to solve to determine the solution of the system.

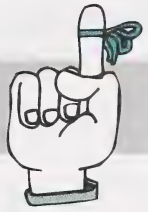


If you need to refresh your memory on how to solve a linear equation, turn to page 442 of the textbook and read the entry "solve a linear equation." Work through the sample calculations.

**2.** Answer exercises 3.b., 3.c., and 4.c. of "Practise Your Prior Skills" on page 131 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 3, page 87.**

**Remember:** You can solve a single equation in two variables if you know the value of one of them.



## Example

Solve for $w$ in $5w+\frac{1}{2}z=46$ if $z=18$.

### Solution

$$5w+\frac{1}{2}z=46$$

$5w+\frac{1}{2}(18)=46$ ← Substitute 18 for $z$.

$5w+9=46$ ← Simplify.

$5w=35$ ← Subtract 9 from both sides.

$w=7$ ← Divide both sides by 5.

**3.** Determine the value of $x$ in each of the following given that $y=-2$.

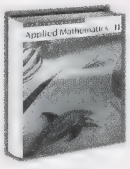
**a.** $3x+5y=5$ **b.** $y-12=7x-2$

**Compare your responses with the suggested answers in the Appendix, Activity 3, page 87.**



## SUBSTITUTION—VARIABLE ALREADY ISOLATED

The method of solving a system by substitution is easiest when **one** of the equations has a variable isolated on one side of the equation. That makes it easy to see what to substitute into the other equation.



Turn to pages 132 and 133 of the textbook and work through "Example 1: Obtain an approximate solution by graphing and an exact solution by substitution."

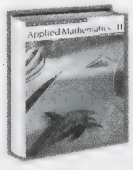


Notice that the equations are labelled equation 1 and equation 2. These labels allow you to keep track of how the equations are used in the solution.

**4.** Answer exercises 2.a., 4, and 5 of "Exercises: Checking Your Skills" on pages 136 and 137 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 3, pages 88–90.**

In some cases, the part from equation (1) substituted into equation (2) is more than just a number. The part from equation (1) that is substituted may be a number and a variable.



Turn to page 134 of the textbook and work through "Example 2: Solve a system of linear equations by substitution." In this example, equation (2) has the variable $f$ isolated on the left side of the equation.

**5.** Answer exercise 3 of "Discussing the Ideas" on page 136 of the textbook.

**6.** Answer exercises 2.c., 6, and 7 of "Exercises: Checking Your Skills" on pages 136 and 137 of the textook.

**Compare your responses with the suggested answers in the Appendix, Activity 3, pages 90–93.**

## SUBSTITUTION—VARIABLE NOT YET ISOLATED

The method of solving a system by substitution is a little more involved when neither equation has a variable isolated.




Turn to pages 135 and 136 of the textbook and work through "Example 3: Solve a system of linear equations by substitution."



A table helps to organize the information and to model the situation algebraically.

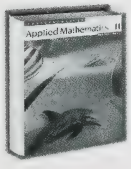
**7.** Answer exercises 3.a., 3.d., 8, 11, 12, and 14 of "Exercises: Checking Your Skills" on pages 137 to 139 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 3, pages 94–101.**

## LOOKING BACK



In this activity you solved systems of linear equations by substitution. You also solved problems involving systems of equations.




**8.** Turn to page 139 of the textbook and answer "Communicating the Ideas."

**Compare your response with the suggested answer in the Appendix, Activity 3, pages 102–103.**

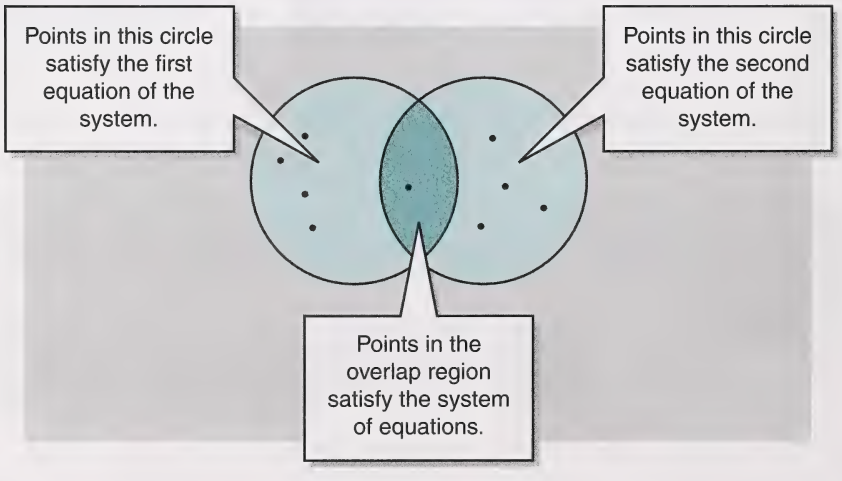
# ACTIVITY 4



## Properties of Linear Systems

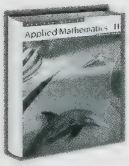
When a group of people meet to solve a problem, they may decide upon one solution, no solution, or several solutions. Likewise, when solving a system of linear equations, you will find that there may be one solution, no solution, or several solutions.

The following Venn diagram shows how the points that satisfy individual equations of a system of equations compare to the points that satisfy the system of equations. For some systems, the overlap region may not have any points; for others, there may be many.



The linear systems you have investigated so far have had only one solution. A Venn diagram would represent such systems with only one point in the overlap region.

For other systems, the overlap region may have many points or none at all. In this activity you will identify linear systems that have zero, one, or an infinite number of solutions; and you will discover ways of altering systems of equations without affecting their solution.

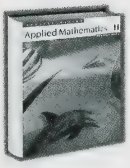



Turn to page 142 of the textbook and read the introductory paragraphs of Tutorial 3.4, "Properties of Linear Systems."

To investigate the number of solutions of a system of equations, you will need to enter a program, called LINES, into your graphing calculator. This program will make graphing more convenient.

Turn to page 422 of the textbook and refer to "LINES" of "Utility 24: TI-83 Programs." Read the first paragraph; then set up your graphing calculator as described. If you are having trouble entering this program, refer to the TI-83 Programs section in the Appendix of this Student Module Booklet and follow the keystrokes given.

## FINDING THE NUMBER OF SOLUTIONS




By solving systems by graphing, you can see whether a system has zero, one, or an infinite number of points satisfying it.

You may want to test the LINES program on a system of linear equations that you have already graphed. For example, graph the system in exercise 3.a. on page 121 of the textbook.

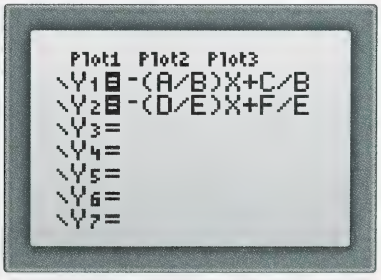
$x + y = 5$ (1)

$3x + y = 3$ (2)

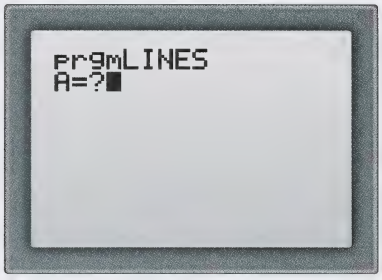
Before using the LINES program, recall that you must enter the following equations into the equation editor.

Y= (−) ( ALPHA [A] ÷ ALPHA [B] ) X,T,Θ,n + ALPHA [C] ÷ ALPHA [B] ENTER

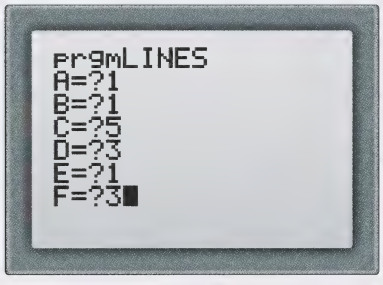
(−) ( ALPHA [D] ÷ ALPHA [E] ) X,T,Θ,n + ALPHA [F] ÷ ALPHA [E]

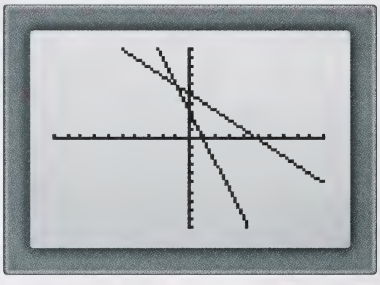


Start the program LINES by pressing PRGM and selecting "LINES." Then press ENTER. Your display should look like the following.




The program now prompts you to enter the values of $A$ to $F$ in the equations $Ax + By = C$ and $Dx + Ey = F$. Enter the values for each unknown, pressing ENTER after each entry. The graph should look like the following.

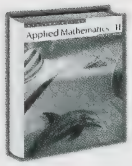


The display here shows the graphs on the standard window settings. Press ZOOM 6 to display your graph on these settings.

Is your program working okay? If you have to edit the program code, press PRGM ▶ (selecting the EDIT menu) and select "LINES." Make the necessary program corrections; then press 2nd [ QUIT ] to get out of the program.





1. Turn to page 142 of the textbook and answer all parts of exercise 1 of "Investigation 1: The Number of Solutions of a Linear System."

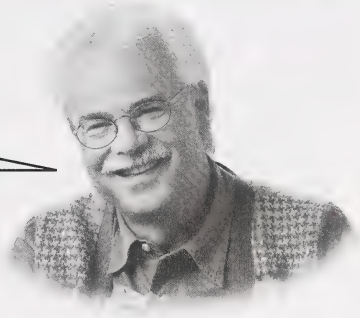
2. List four points that satisfy the system of equations in exercise 1 of "Investigation 1: The Number of Solutions of a Linear System."

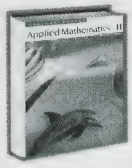
3. Turn to pages 142 and 143 of the textbook and answer exercises 2 and 3 of "Investigation 1: The Number of Solutions of a Linear System."

You may want to review how to use the Intersect feature to find the intersection point of two lines. See Activity 1.

**Compare your responses with the suggested answers in the Appendix, Activity 4, pages 104–108.**

Mathematicians have a particular term for linear systems that have one solution or an infinite number of solutions and another term for linear systems that have no solution.





Turn to page 143 of the textbook and read the information following Investigation 1.

4. Answer exercises 3 and 5 of "Exercises: Checking Your Skills" on page 147 of the textbook.

5. Refer to your answers to exercise 5 of "Exercises: Checking Your Skills." Rearrange each equation in slope-intercept form, $y = mx + b$.

   **a.** How do the values of $m$ (the coefficients of $x$) and the values of $b$ compare in the systems for no solution, for one solution, and for infinitely many solutions.

   **b.** How can you use $m$ and $b$ to identify inconsistent systems?

**Compare your responses with the suggested answers in the Appendix, Activity 4, pages 108–110.**

Up to this point, you have analysed how to determine if a linear system has one solution, no solution, or an infinite number of solutions. In the next example, you will determine if a particular point is a solution of a linear system.



Turn to page 146 of the textbook and work through "Example: Check to determine if a point is a solution of a system."

6. Answer exercises 1, 2.b., 2.c., 4.a., and 4.b. of "Exercises: Checking Your Skills" on page 147 of the textbook.

**Compare your responses with the suggested answers in the Appendix, Activity 4, pages 110–112.**

## CHANGING EQUATIONS IN A LINEAR SYSTEM

What changes can be made to linear systems without affecting their solutions? If such changes exist, algebraic methods can be developed to solve linear systems.



7. Turn to page 144 of the textbook and answer exercises 1 to 6 of "Investigation 2: Properties of Linear Systems."

8. Turn to pages 144 and 145 of the textbook and answer exercises 1 to 7 of "Investigation 3: Adding or Subtracting the Equations of a Linear System."

9. Turn to page 148 of the textbook and answer exercise 7 of "Exercises: Checking Your Skills."

**Compare your responses with the suggested answers in the Appendix, Activity 4, pages 113–116.**

## LOOKING BACK

In this activity, you identified a linear system of equations with one point in its solution, with an infinite number of points in its solution, and with no points in its solution. You discovered that a linear system is consistent when it has one or more points in its solution and that a linear system is inconsistent when it has no points in its solution. You then modified linear systems by replacing an equation by a non-zero multiple of itself $(2x+3y=5$ replaced by $6x+9y=15)$. You also modified linear systems by replacing an equation by a sum or difference of itself and another equation from the system. In both cases, the solutions to the linear systems remained the same.



10. Turn to page 148 of the textbook and answer "Communicating the Ideas."

**Compare your response with the suggested answer in the Appendix, Activity 4, page 116.**

# ACTIVITY 5



# Solving Linear Systems by Elimination

Besides the method of substitution, there is another algebraic method of solving systems of equations, called elimination. You have learned that certain changes can be made to a system without affecting its solution. The method of elimination is based on such changes.

**Example**

Travelling upstream, a boat can cruise at 60 km/h. Downstream, in the same river, the boat can cruise at 65 km/h. How fast does the boat cruise in still water?

## Solution

Let $x$ be the speed (in km/h) of the boat in still water. Let $y$ be the speed (in km/h) of the current.

Therefore, the speed of the boat upstream is $x - y$ and the speed of the boat downstream is $x + y$.

The situation can be modelled by the following linear system:

$$\begin{aligned} x - y &= 60 && (1)\\ x + y &= 65 && (2)\\ \hline 2x + 0y &= 125 && (1) + (2) \quad \leftarrow \text{Add equations (1) and (2).}\\ 2x &= 125\\ x &= 62.5 \end{aligned}$$

The boat cruises at a speed of 62.5 km/h in still water.

## ADDING AND SUBTRACTING EQUATIONS

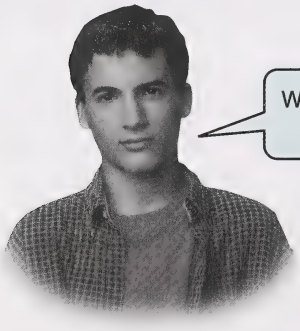



The method of elimination is based on a property of linear systems, where equations may be added or subtracted without changing the solution of the linear system.

Turn to page 149 of the textbook and read the introduction to Tutorial 3.5, "Solving Linear Systems by Elimination." Then work through "Example 1: Solve a linear system by addition" on pages 149 and 150.



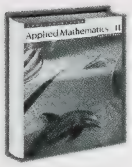
The example shows that if the coefficients for at least one of the variables in the system are the same but opposite in sign, you **add** the equations to eliminate the variable.



What if the coefficients of a variable are the same and not opposite in sign?

Then you **subtract** the equations to eliminate the variable.

## Example




Answer exercise 1.c. of "Exercises: Checking Your Skills" on page 153 of the textbook.

### Solution

$3x-4y=0$ (1)
$5x-4y=8$ (2)

The coefficients of $y$ are the same, $-4$. To eliminate $y$, the equations must be subtracted.

Subtract equation (2) from equation (1), and solve for $x$.

$$\begin{array}{rl} 3x-4y=\ 0 & (1) \\ 5x-4y=\ 8 & (2) \\ \hline -2x+0y=-8 & (1)-(2) \\ -2x=-8 & \leftarrow \text{Simplify.} \\ x=4 & \leftarrow \text{Divide both sides by } -2. \end{array}$$

$$\begin{array}{l} 3x-5x=-2x \\ -4y-(-4y)=0 \\ 0-8=-8 \end{array}$$

Substitute 4 for $x$ into one of the original equations to find the value of $y$.

$$\begin{aligned} 3x-4y&=0 \\ 3(4)-4y&=0 \\ 12-4y&=0 \\ -4y&=-12 \\ y&=3 \end{aligned}$$

Check if $(4,3)$ is a solution.

| LS | RS |
|---|---|
| $3x-4y$ | 0 |
| $=3(4)-4(3)$ | |
| $=12-12$ | |
| $=0$ | |

LS = RS

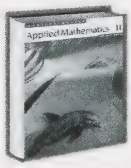
| LS | RS |
|---|---|
| $5x-4y$ | 8 |
| $=5(4)-4(3)$ | |
| $=20-12$ | |
| $=8$ | |

LS = RS

The ordered pair $(4,3)$ is a solution.

Remember to number equations in the original system and to complete your answer to word problems in sentence form.





1. Turn to page 153 of the textbook and answer exercise 1 of "Discussing the Ideas."

2. Turn to pages 153 and 154 of the textbook and answer exercises 1.b., 3, and 4 of "Exercises: Checking Your Skills."

**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 117–119.**

## MULTIPLYING EQUATIONS BY CONSTANTS

The coefficients of a variable may be neither equal nor opposite. In such cases, multiplying one of the equations by a constant can give you an equivalent system—one that can be solved by subtraction or addition.



Turn to pages 150 and 151 of the textbook and work through "Example 2: Multiply one equation by a constant, then subtract to eliminate."

3. Answer exercises 2 and 3 of "Discussing the Ideas" on page 153 of the textbook.

4. Answer exercises 2.b., 2.c., and 5 of "Exercises: Checking Your Skills" on pages 153 and 154 of the textbook.

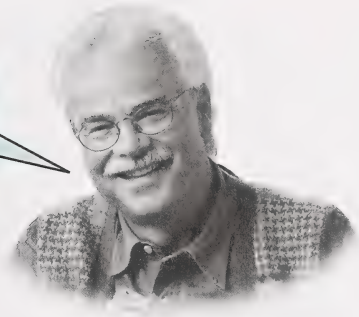
**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 119–121.**



For some systems, you may wish to multiply both equations by a different, suitable constant.

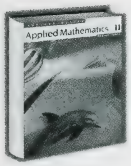


Turn to pages 151 and 152 of the textbook and work through "Example 3: Multiply each equation by an appropriate number, then add or subtract to eliminate a variable."



Notice how the table was used to organize the information. Make the new coefficients the same as the lowest common multiple.

After multiplying the equations by suitable constants, notice that the new equations have coefficients that are the same for one of the variables. In other linear systems of equations that you will work with, the coefficients will be the same in numerical value but opposite in sign. In these linear systems you need to add to eliminate the variable.




**5.** Answer exercises 4, 5, and 6 of "Discussing the Ideas" on page 153 of the textbook.

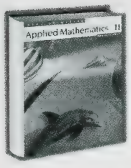
**6.** Solve the following system of equations using the method of elimination.

$$2x+5y=39 \quad (1)$$
$$3x-4y=-22 \quad (2)$$

**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 122–123.**

## APPLICATIONS

In applying the method of elimination to real-world problems, you will first have to model the problem algebraically as a system of equations. You may want to review Activity 2: Algebraic Modelling. Recall that rewording, tabulating information, and drawing diagrams are often helpful in algebraic modelling.




**7.** Turn to pages 154 to 156 of the textbook and answer exercises 6, 7, and 11 of "Exercises: Checking Your Skills."

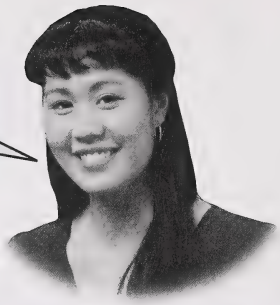
Watch signs when subtracting with negative numbers. For example,

- $23-(-5)=28$, not 18
- $-7-(-2)=-5$, not 5 or $-9$



**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 123–126.**






**8.** Turn to page 157 of the textbook and answer exercises 12 and 13 of "Exercises: Extending Your Thinking."

**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 126–127.**

## LOOKING BACK

In this activity you solved systems of linear equations by elimination. You then used the elimination method to solve linear systems of equations involving practical problems.

**9.** Substitution and elimination are algebraic methods of solving a linear system.

**a.** Make up one system of equations that you can solve fairly easily using the substitution method or the elimination method.

**b.** Solve the system using both methods.

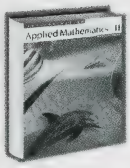
**c.** Indicate which method you think is better for your system. Explain.

**Compare your responses with the suggested answers in the Appendix, Activity 5, pages 127–128.**



# Follow-up Activities

This module dealt with Chapter 3: Linear Systems in the *Addison-Wesley Applied Mathematics 11 Source Book.*



Turn to the table on page 160 of the textbook and take note of the skills and concepts listed for each of the tutorials. Also, read the column of important results and formulas you discovered. The table indicates concisely what you should know and be able to do after participating in the learning activities of this module.

Answer exercises 1.a. to 1.h. of Part A of "What Should I Be Able To Do?" on page 161 of the textbook. For exercise 1.e., if you used your graphing calculator, use the Intersect feature from the CALCULATE menu. This question provides an opportunity for you to apply several concepts and skills at the same time.

**Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 129–131.**

If you had difficulties understanding the skills and concepts in Module 3: Linear Systems, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

## EXTRA HELP

Maybe you found the method of elimination difficult. If so, remembering these general steps may help you with this method.

### Elimination Method

**Step 1:** Express both equations in $Ax + By = C$ form. If the equations are already in this form, skip this step.

**Step 2:** Choose either $x$ or $y$ as the variable to be eliminated. Then, if necessary, multiply one or both equations so the coefficients of the variable chosen for elimination are equal or opposite.

**Step 3:** If Step 2 provided equal coefficients, subtract the equations. If it provided opposite coefficients, add the equations.

**Step 4:** Solve the equation you obtained in Step 3. It should only be in one variable. (The variable you picked for elimination should have disappeared.)

**Step 5:** Substitute the value of the variable you calculated in Step 4 into one of the original equations of the system, and solve for the remaining variable.

**Step 6:** Check if the values are a solution to the system. Make sure the solution satisfies both original equations.

**Step 7:** State the solution.

## Example

Solve the following system of equations using the elimination method:

$$11y = -10x + 14 \quad (1)$$
$$14x + 94 = 13y \quad (2)$$

### Solution

**Step 1: Express both equations in $Ax + By = C$ form.**

$$11y = -10x + 14$$
$$11y + 10x = -10x + 14 + 10x$$
$$10x + 11y = 14 \quad (3)$$

$$14x + 94 = 13y$$
$$14x + 94 - 94 = 13y - 94$$
$$14x = 13y - 94$$
$$14x - 13y = 13y - 94 - 13y$$
$$14x - 13y = -94 \quad (4)$$

**Step 2: Choose either $x$ or $y$ as the variable to be eliminated. Then, if necessary, multiply one or both equations so the coefficients of the variable chosen for elimination are equal or opposite.**

Eliminate the variable $x$ because it will be easier to find the lowest common multiple (LCM) of 10 and 14 than finding the LCM of 11 and 13.

The LCM of 10 and 14 is $70 (10 \times 7 = 70$ and $14 \times 5 = 70)$. It is the smallest number that both coefficients can divide into evenly. So, multiply equation (3) by 7 and equation (4) by 5.

$$70x + 77y = 98 \quad (5): 7 \times (3)$$
$$70x - 65y = -470 \quad (6): 5 \times (4)$$

**Step 3: If Step 2 provided equal coefficients, subtract the equations. If it provided opposite coefficients, add the equations.**

Because the coefficients of $x$ are equal, subtract the equations.

$$\begin{aligned} 70x+77y &= 98 \quad (5)\\ 70x-65y &= -470 \quad (6)\\ \hline 142y &= 568 \quad (5)-(6) \end{aligned}$$

$77y-(-65y)=77y+65y$

$98-(-470)=98+470$

**Step 4: Solve the equation you obtained in Step 3.**

$$\begin{aligned} 142y &= 568\\ \frac{142y}{142} &= \frac{568}{142}\\ y &= 4 \end{aligned}$$

**Step 5: Substitute the value of the variable you calculated in Step 4 into one of the original equations.**

Substitute 4 for $y$ into equation (2).

$$\begin{aligned} 14x+94 &= 13y\\ 14x+94 &= 13(4)\\ 14x+94 &= 52\\ 14x+94-94 &= 52-94\\ 14x &= -42\\ \frac{14x}{14} &= \frac{-42}{14}\\ x &= -3 \end{aligned}$$

**Step 6: Check if the values are a solution to the system.**

Substitute $-3$ for $x$ and 4 for $y$ into both original equations.

| LS | RS |
|---|---|
| $11y$ | $-10x+14$ |
| $=11(4)$ | $=-10(-3)+14$ |
| $=44$ | $=30+14$ |
| | $=44$ |

LS = RS

| LS | RS |
|---|---|
| $14x+94$ | $13y$ |
| $=14(-3)+94$ | $=13(4)$ |
| $=-42+94$ | $=52$ |
| $=52$ | |

LS = RS

**Step 7: State the solution.**

The solution is $x = -3$ and $y = 4$, or $(-3, 4)$.

Solve the following system of equations by elimination. **Remember:** Check your work.

$$5x + 6y = 13 \quad (1)$$
$$7x + 4y = 5 \quad (2)$$

**Compare your response with the suggested answer in the Appendix, Follow-up Activities, Extra Help, pages 131–132.**

## ENRICHMENT

You probably found that solving systems of equations with the methods of substitution and elimination involves a considerable amount of algebraic manipulation; therefore, it is easy to make an error. Having your calculator doing some of the algebraic work is what this Enrichment is about.

The calculator method you are about to learn involves entering the coefficients and the constants of a system of linear equations into rectangular arrays of numbers. Each rectangular array is called a **matrix**.

Look at the following system of equations:

$$x + 4y = 6 \quad (1)$$
$$2x - 3y = 1 \quad (2)$$

This linear system can be represented by two matrices: one for the coefficients on the left side of the equal sign and one for the constants on the right side. Let the matrix formed from the coefficients be $A$ and the matrix formed from the constants be $C$.

Matrix $A$ for this system is formed from the coefficients of the variables.

$$\begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$$

Matrix $C$ is formed from the constants on the right side of the equal signs.

$$\begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

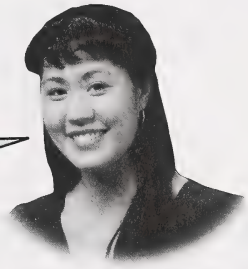
For this system, matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$ and matrix $C = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$.

Let matrix $X$ be a certain product: $X = A^{-1}C$, where $A^{-1}$ is the "multiplicative inverse" of matrix $A$. That is, matrix $X$ is the "product" of matrix $A^{-1}$ and matrix $C$. Here, the operation of multiplication is not the familiar kind. Fortunately, you can leave the details of this operation to your graphing calculator—it will display matrix $X$ for you. It turns out that the entries of matrix $X$ correspond to values of variables $x$ and $y$ that satisfy the system of equations.



How does matrix multiplication get me the solution to the system?

In the following example you will see how to use your calculator to determine the matrix value for $X$. From that, you will find the solution of the original system of equations



## Example

Solve the following using matrix operations on your graphing calculator:

$x + 4y = 6$ (1)
$2x - 3y = 1$ (2)

### Solution

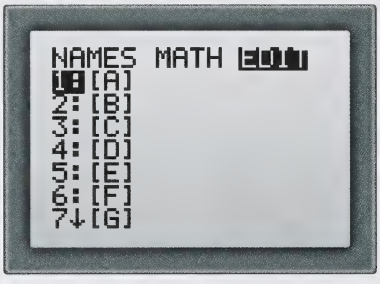
Before starting, clear any matrices in your calculator's memory. Press [2nd] [ MEM ] [2] (2:Delete…) [5] (5:Matrix…). Then press [ENTER] repeatedly until there are no matrices listed.

**Step 1:** Write the system of equations with the variables on the left and in corresponding positions and with the constants on the right.

$$x + 4y = 6 \quad (1)$$
$$2x - 3y = 1 \quad (2)$$

**Step 2:** Enter matrix *A*.

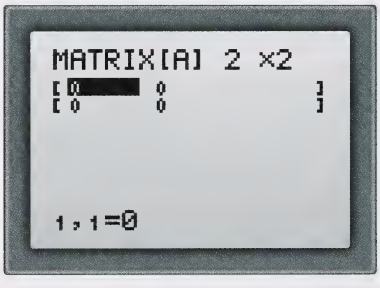
- Press (MATRX) (▶) (▶) (selecting the EDIT menu) (1) to select matrix A.





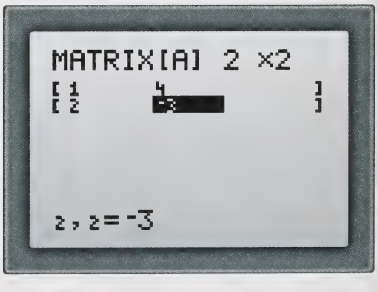


- Change the expression "1×1" at the top of the screen to "2×2." This indicates that there are 2 rows and 2 columns in this matrix.



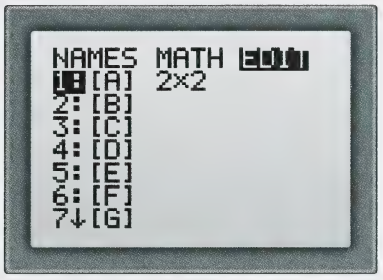

- Enter the elements of the matrix going across the first row and then across the second row.

(1) (ENTER) (4) (ENTER)
(2) (ENTER) (–) (3)
(ENTER)



**Step 3:** Enter matrix *C*.

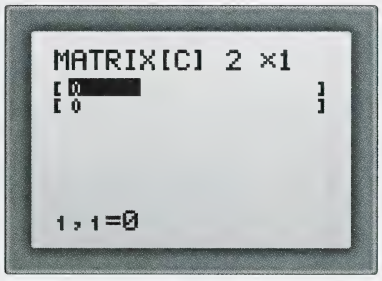
- Press MATRX ▶ ▶ (selecting the EDIT menu) 3 to select matrix *C*.





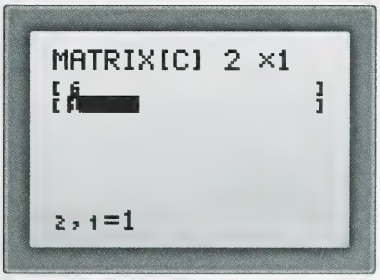


- Change the expression "1×1" at the top of the screen to "2×1." This indicates that there are 2 rows and 1 column in this matrix.




- Enter the two elements of the matrix.

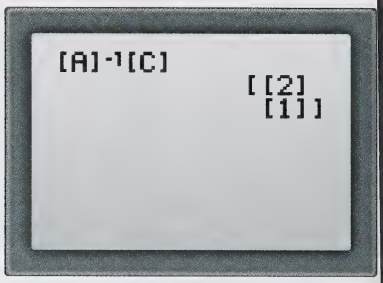
6 ENTER 1 ENTER




- Press 2nd [ QUIT ] to quit the entry of *C*.

**Step 4:** Determine the value of matrix $X$ by calculating $A^{-1}C$.

MATRX 1 $x^{-1}$ MATRX 3
ENTER




Matrix $X$ is equal to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

The $x$-value is indicated by the top entry of the matrix, and the $y$-value is indicated by the bottom entry. Therefore, $x = 2$ and $y = 1$, or $(2, 1)$.

**Step 5:** Check if $x = 2$ and $y = 1$ is a solution of the system.

| LS | RS |
|---|---|
| $x + 4y$ | 6 |
| $= (2) + 4(1)$ | |
| $= 2 + 4$ | |
| $= 6$ | |

LS = RS

| LS | RS |
|---|---|
| $2x - 3y$ | 1 |
| $= 2(2) - 3(1)$ | |
| $= 4 - 3$ | |
| $= 1$ | |

LS = RS

The solution is $(2, 1)$.



Turn to page 127 of the textbook and solve the system of equations developed in "Example 3: Use a table to organize information."

**Compare your response with the suggested answer in the Appendix, Follow-up Activities, Enrichment, pages 132–133.**

You may want to solve other systems of equations with matrices. Try some of the systems from this module that you solved earlier by graphing, by substitution, or by elimination.

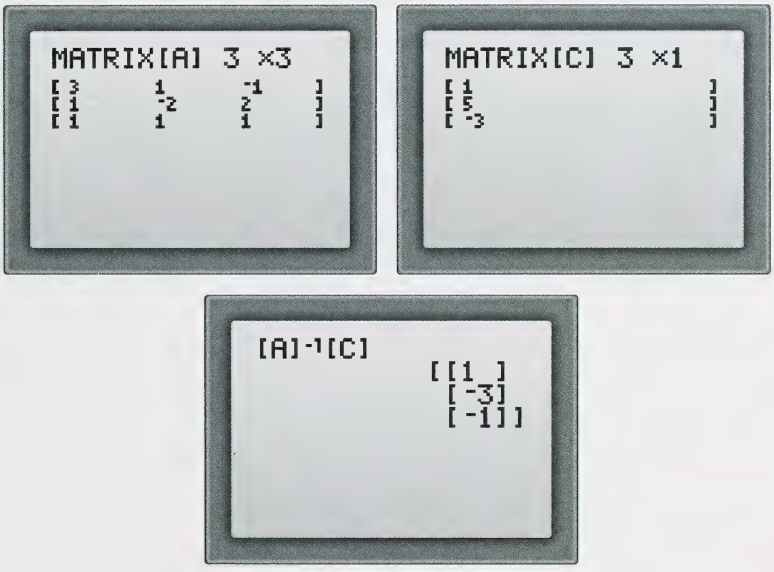
So far, you have solved systems of two equations in two variables with matrices. Larger systems can also be solved using larger matrices. For a system of three equations in three variables, the matrix of the coefficients, which you named $A$, has three rows and three columns. The matrix of constants for a system of three equations, which you named $C$, has three rows, and one column.

## Example

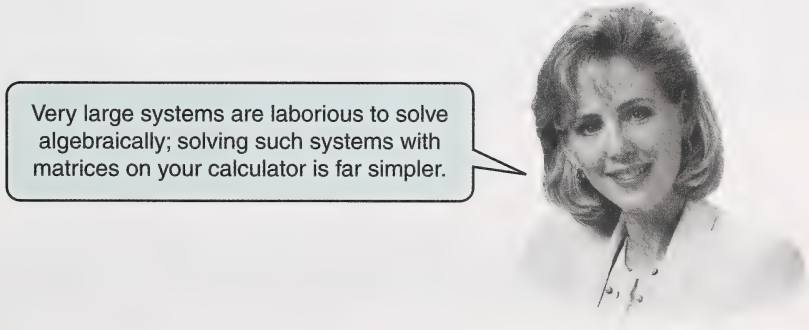
Find the solution to the following system of equations:

$$3x + y - z = 1 \quad (1)$$
$$x - 2y + 2z = 5 \quad (2)$$
$$x + y + z = -3 \quad (3)$$

### Solution




The solution is $x = 1$, $y = -3$, and $z = -1$, or $(1, -3, -1)$.



# MODULE PROJECT
## SKYDIVING



## Completing the Project

At the beginning of this module, you were introduced to the requirements for your Module 3 project, Skydiving. You should now have done a significant part of your research for this project.



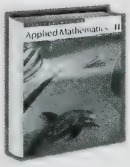
Turn to page 140 of the textbook and read the beginning of "Deploying a Parachute."

1. Explain why a large snowflake falls very slowly.

2. Answer exercises 1 and 2 of "Deploying a Parachute" on pages 140 and 141 of the textbook. **Note:** If possible, do exercise 2 with a partner; otherwise, complete it on your own.

Later, in completing the project booklet, you will repeat exercises 1 and 2 for a skydiver in the dive position rather than in the spread stable position.

**Compare your responses with the suggested answers in the Appendix, Module Project, pages 133–136.**

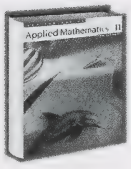



Turn to pages 158 and 159 of the textbook and preview "Skydiving to Terminal Velocity." Pay special attention to exercises 1, 2, and 3. Then turn to pages 163 to 166 of the textbook and read Part C of "What I Should be Able To Do?" The student work on pages 164 and 165 are based mostly on exercise 3 of "Skydiving to Terminal Velocity" on page 159. Exercises 1 and 2 on page 158 show how to determine the data for free fall before reaching terminal velocity.

The following table may help you in understanding how the student calculated the data for the parachute jumps.

| Stages of Jump | Skydiver Using Dive Position | Skydiver Using Spread Stable Position |
|---|---|---|
| 1. free fall before reaching terminal velocity (skydiver is accelerating) | Height is based on $h = -4.9t^2 + 3600$<br><br>Downward speed: $v = 9.8t$<br><br>Duration is the value of $t$ in the solution of this system:<br><br>$h = -4.9t^2 + 3600$ (1)<br><br>$h = 3600 - 367.8$<br>$h = 3232.2$ (2) | Height is based on chart on page 159. The chart distances are subtracted from 3600. (Student chose 3600 for *b*.)<br><br>Duration is given as 12 s. (page 158 of textbook0 |
| 2. free fall at terminal velocity (skydiver is descending with constant velocity) | Downward speed is 85 m/s (rounded from 84.9 m/s).<br><br>Height is based on $h = -85t + 3232.2$, where $t$ is the elapsed time (in s) within this stage and 3232.2 m is the height at which this stage starts. | Downward speed is 53 m/s (rounded from 52.8m/s).<br><br>Height is based on $h = -53t + 3147.5$, where $t$ is the elapsed time (in s) within this stage and 3147.5 m $(3600 - 452.5)$ is the height at which this stage starts. |

| | | |
|---|---|---|
| 3. fall with parachute open (skydiver is descending with constant velocity) | Downward speed is 4 m/s.<br><br>Height is based on $h = -4t + 823.3$, where $t$ is the elapsed time (in s) within this stage and 823.3 m is the height at which this stage starts. | Downward speed is 4 m/s.<br><br>Height is based on $h = -4t + 762.5$, where $t$ is the elapsed time (in s) within this stage and 762.5 m is the height at which this stage starts. |




Refer to the sample project again on pages 164 to 166 of the textbook.



**3.** Do the heights look reasonable? To support your answer, calculate the heights for at least six different times. Include two unique times within each stage of the jump.

**4.** On the tables for dive and spread stable positions, are the data for the three stages of the jump identified correctly?

**5.** Notice that the portion of the graphs corresponding to the first stage is curved. Why are the portions of the graphs that correspond to the second and third stages straight, rather than curved?

**Compare your responses with the suggested answers in the Appendix, Module Project, pages 136–138.**

## The Module Project



Now that you have previewed the exercises on pages 158 and 159 and analysed the sample student work on pages 164 to 166, take out the Project Booklet that accompanies this Student Module Booklet and complete the module project, Skydiving.

**Submit your completed Module 3 Project Booklet to your teacher.**

# Module Summary

In this module, you modelled problem situations using systems of linear equations. You solved linear equations by graphing and by using the algebraic methods of substitution and elimination. Throughout the module, you used solutions of linear systems to solve real-world problems.

In your module project, you examined how linear equations can be used to determine distances of free fall and heights in parachuting. You were shown how the parachute is used for recreational purposes.


REUTERS

The parachute is also used for important missions, or tasks. Antarctica is the continent surrounding the South Pole. In November 1998, Jerri Nielson, an American doctor, went to the remote Amundsen-Scott South Pole Research Station in Antarctica to serve as the medic for a one-year period. After the start of the Antarctic winter, she discovered a lump in one of her breasts; it turned out to be breast cancer. During the extremely cold winter weather conditions, air evacuation was out of the question. With no medication to treat her condition, it appeared that she faced a critical delay of months before she could get proper medical treatment. Although difficult, cargo containing the medication for chemotherapy was dropped to the research station by parachute from an airplane flying overhead. This medication allowed Dr. Nielson to begin chemotherapy.

Several months later, with improved weather, she was airlifted from the isolated research station. When she returned to the United States, she started full medical treatment for her condition; but it was the earlier airdrop that first saved her life.



## Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

**Submit your completed Module 3 Assignment Booklet to your teacher.**

# APPENDIX

## Glossary
## Suggested Answers
## T1-83 Programs
## Image Credits

## GLOSSARY

**break-even point:** the point at which a business venture operates so there is neither a profit nor a loss

**elimination:** a process in which the equations of a system are combined in such a way that one of the variables is removed

**linear equation:** an equation whose graph is a straight line; an equation that can be written in the form $y = mx + b$

**linear system:** two or more linear equations in the same variables

**matrix:** a rectangular arrangement of numbers in rows and columns

**ordered pair:** a pair of numbers that represent a point on the coordinate plane

**solution of a system of equations:** an ordered pair that satisfies all the equations of a system

**substitution:** a process in which an equivalent expression is substituted for a variable in an equation so that the original variable is removed

***x*-intercept:** the coordinates of a point where a graph crosses the horizontal axis

***y*-intercept:** the coordinates of a point where a graph crosses the vertical axis

## SUGGESTED ANSWERS

# Activity 1: Solving Linear Systems by Graphing

**1. Textbook exercise 1 of "Exercises: Checking Your Skills," p. 121**

**1. a.** $5x + 6y = 1$ (1)
$6x + 2y = -3$ (2)

Check if $(-1, 1)$ is a solution.

| LS | RS |
|---|---|
| $5x + 6y$ | 1 |
| $= 5(-1) + 6(1)$ | |
| $= -5 + 6$ | |
| $= 1$ | |

LS = RS

| LS | RS |
|---|---|
| $6x + 2y$ | $-3$ |
| $= 6(-1) + 2(1)$ | |
| $= -6 + 2$ | |
| $= -4$ | |

LS ≠ RS

The point $(-1, 1)$ satisfies equation (1), but not equation (2). Therefore, $(-1, 1)$ is not a solution of the system.

**b.** $3x+4y=1$ ①
$5x-3y=-8$ ②

Check if $(-1,1)$ is a solution.

| LS | RS |
|---|---|
| $3x+4y$ | 1 |
| $=3(-1)+4(1)$ | |
| $=-3+4$ | |
| $=1$ | |

LS = RS

| LS | RS |
|---|---|
| $5x-3y$ | $-8$ |
| $=5(-1)-3(1)$ | |
| $=-5-3$ | |
| $=-8$ | |

LS = RS

The point $(-1,1)$ satifies both equations and is, therefore, a solution of the system.

**c.** $7x-3y=10$ ①
$6x+5y=-1$ ②

Check if $(-1,1)$ is a solution.

| LS | RS |
|---|---|
| $7x-3y$ | 10 |
| $=7(-1)-3(1)$ | |
| $=-7-3$ | |
| $=-10$ | |

LS ≠ RS

| LS | RS |
|---|---|
| $6x+5y$ | $-1$ |
| $=6(-1)+5(1)$ | |
| $=-6+5$ | |
| $=-1$ | |

LS = RS

The point $(-1,1)$ does not satisfy equation ①. Therefore, the point $(-1,1)$ does not satisfy the system.

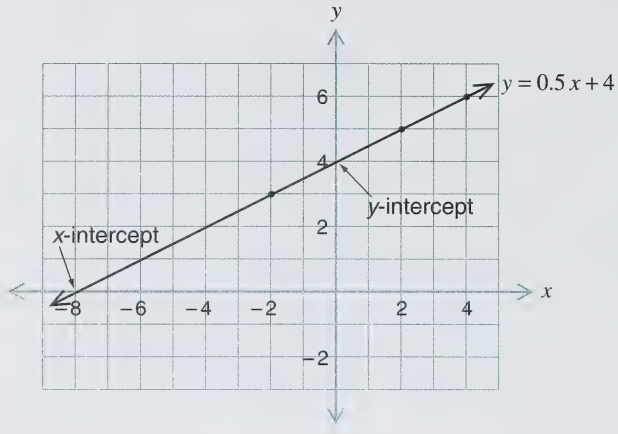
The point $(-1,1)$ is a solution of the system in 1.b only.

## Activity 1 (continued)

**2. Textbook exercises 1 to 5 of "Practice Your Prior Skills," p. 114**

**1.** $y = 0.5x + 4$

| $x$ | $y$ |
|---|---|
| $-2$ | 3 |
| 2 | 5 |
| 4 | 6 |




According to the graph, the $x$-intercept is $-8$ and the $y$-intercept is 4.

**2. a.** $2x + 3y - 6 = 0$

$2x + 3y = 6$ ← Add 6 to both sides.

$3y = -2x + 6$ ← Subtract $2x$ from both sides.

$y = -\frac{2}{3}x + 2$ ← Divide both sides by 3.

**b.** $5x - 2y + 1 = 0$

$-2y + 1 = -5x$ ← Subtract $5x$ from both sides.

$-2y = -5x - 1$ ← Subtract 1 from both sides.

$y = \frac{5}{2}x + \frac{1}{2}$ ← Divide both sides by $-2$.

**3. a.** $y = 2x - 3$

$m$ $b$

The slope is 2, and the $y$-intercept is $-3$.

**b.** $y = 6 - 4x$

$y = -4x + 6$ ← Rearrange into $y = mx + b$ form.

$m$ $b$

The slope is $-4$, and the $y$-intercept is 6.

**c.** $x + 3y + 15 = 0$

$3y + 15 = -x$

$3y = -x - 15$

$y = -\frac{1}{3}x - 5$

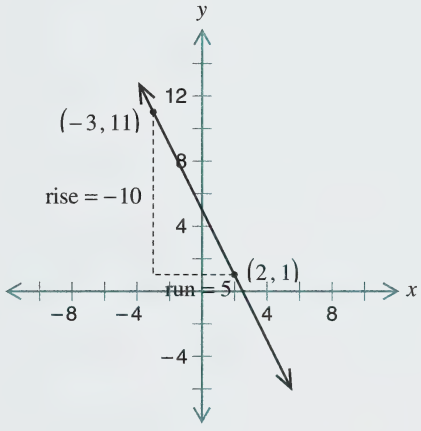
$m$ $b$

The slope is $-\frac{1}{3}$, and the $y$-intercept is $-5$.

**4.** Because the $y$-intercept is $-2$, $b = -2$. Because the slope is $\frac{3}{4}$, $m = \frac{3}{4}$. Therefore, in the form $y = mx + b$, the equation is $y = \frac{3}{4}x - 2$.

**5.** First, find the slope, $m$, of the line.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-10}{5}$$
$$= -2$$




Second, find the $y$-intercept. To do this, substitute $m = -2$ and the coordinates of one of the points given into the slope-intercept form, $y = mx + b$. In this case, choose $(2, 1)$.

$y = mx + b$

$1 = -2(2) + b$

$1 = -4 + b$

$b = 1 + 4$

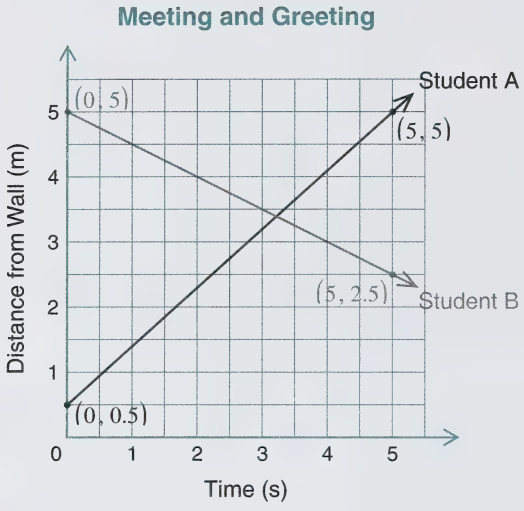
$= 5$

Since $m = -2$ and $b = 5$, the equation of the line is $y = -2x + 5$ or $2x + y - 5 = 0$.

## Activity 1 (continued)

**3. Textbook exercises 4 to 8 of "Investigation 1: Meeting and Greeting," pp. 116 and 117**

**4.**




**5. Student A**

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{5.0 - 0.5}{5 - 0}$$
$$= \frac{4.5}{5}$$
$$= 0.9$$

**Student B**

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2.5 - 5.0}{5 - 0}$$
$$= \frac{-2.5}{5}$$
$$= -0.5$$

The units for the slope are m/s. The slope represents the velocities, both speeds and directions, of the two students.

**6.** Substitute the slope and $y$-intercept of each line into the slope-intercept form, $y = mx + b$.

**Student A**

$m = 0.9$ and $b = 0.5$

$\therefore\ y = mx + b$

$y = 0.9x + 0.5$

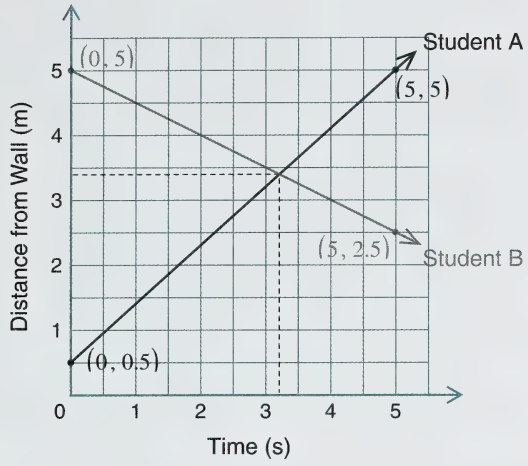
**Student B**

$m = -0.5$ and $b = 5$

$\therefore\ y = mx + b$

$y = -0.5x + 5$

**7.** **Meeting and Greeting**

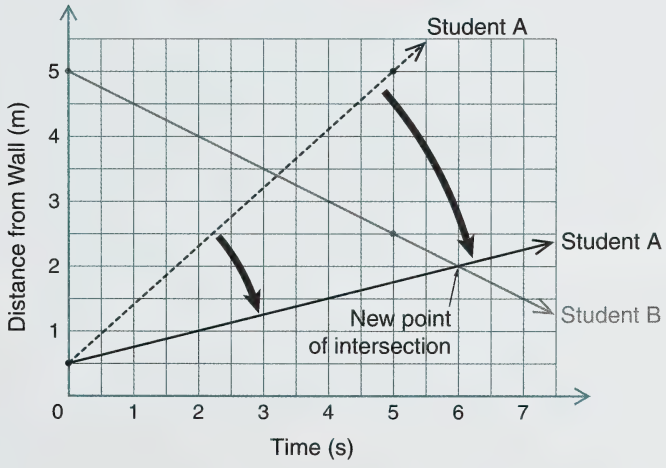



The coordinates of the intersection point are approximately $(3.2, 3.4)$.

**8.** If Student A walked or shuffled much slower than Student B, the slope of the graph for Student A would be smaller. The slope of this graph would be more gradual (less steep).

The intersection point would move down and to the right on the grid. This means the students would meet later and closer to the wall. **Remember:** The distance along the $x$-axis represents time, and the distance along the $y$-axis represents distance from the wall.

**Meeting and Greeting**



## Activity 1 (continued)

**4. Textbook exercises 1 and 2 of "Discussing the Ideas," p. 120**

1. The slope for one student is positive and negative for the other student because the students are moving in opposite directions.

2. The coordinates of the point of intersection indicates when and where the students meet.

**5. Textbook exercise 4 of "Discussing the Ideas," p. 120**

4. The business loses money when the number of slices is less than 400. The business makes money when the number is greater than 400.

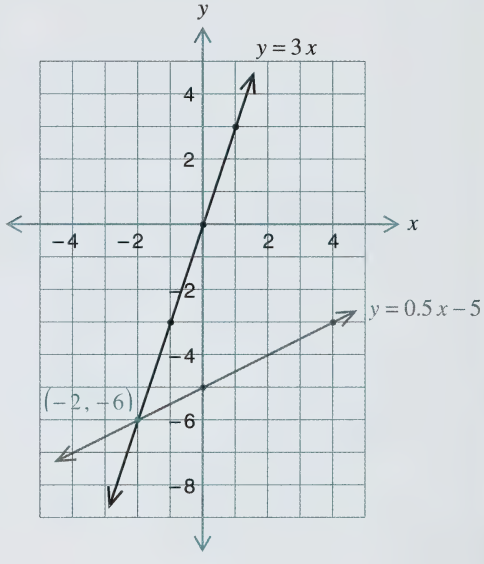
**6. Textbook exercises 2.a., 2.c., 3.a., and 7 of "Exercises: Checking Your Skills," pp. 121 and 122**

**2. a.** $y = 3x$

| $x$ | $y$ |
|---|---|
| $-1$ | $-3$ |
| $0$ | $0$ |
| $1$ | $3$ |

$y = 0.5x - 5$

| $x$ | $y$ |
|---|---|
| $-2$ | $-6$ |
| $0$ | $-5$ |
| $4$ | $-3$ |




Check $(-2, -6)$ in both equations.

| LS | RS |
|---|---|
| $y$ | $3x$ |
| $= -6$ | $= 3(-2)$ |
| | $= -6$ |

LS = RS

| LS | RS |
|---|---|
| $y$ | $0.5x - 5$ |
| $= -6$ | $= 0.5(-2) - 5$ |
| | $= -1 - 5$ |
| | $= -6$ |

LS = RS

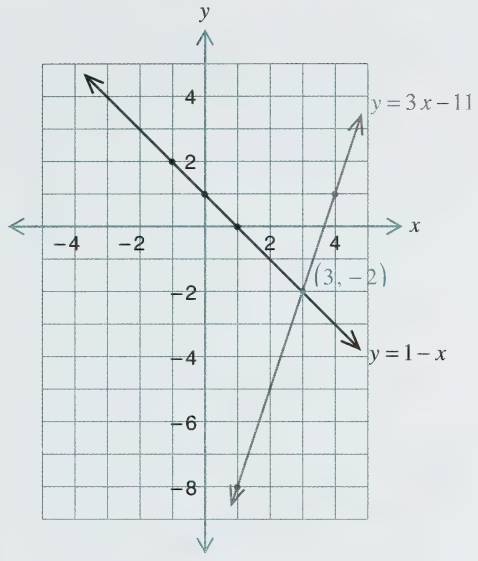
The point $(-2, -6)$ is a solution of the system.

**c.** $y = 1 - x$

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $y$ | $2$ | $1$ | $0$ |

$y = 3x - 11$

| $x$ | $1$ | $3$ | $4$ |
|---|---|---|---|
| $y$ | $-8$ | $-2$ | $1$ |




Check $(3, -2)$ in both equations.

| LS | RS |
|---|---|
| $y$ | $1 - x$ |
| $= -2$ | $= 1 - 3$ |
| | $= -2$ |

LS = RS

| LS | RS |
|---|---|
| $y$ | $3x - 11$ |
| $= -2$ | $= 3(3) - 11$ |
| | $= 9 - 11$ |
| | $= -2$ |

LS = RS

The point $(3, -2)$ is a solution of the system.

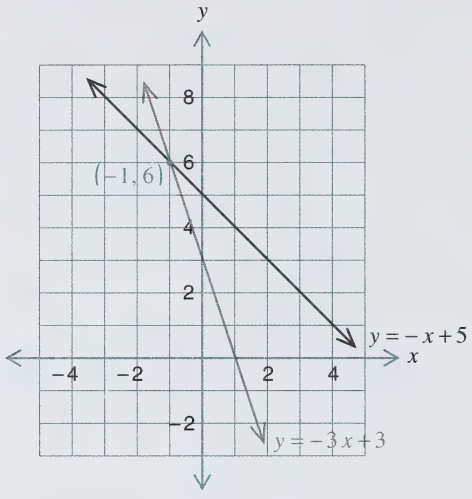
## Activity 1 (continued)

**3. a.** Rearrange the equations into slope-intercept form, $y = mx + b$.

$x + y = 5$ $\qquad$ $3x + y = 3$

$y = -x + 5$ $\qquad$ $y = -3x + 3$

Graph the equations.




Check $(-1, 6)$ for both equations.

| LS | RS |
|---|---|
| $x + y$ | 5 |
| $= -1 + 6$ | |
| $= 5$ | |

LS = RS

| LS | RS |
|---|---|
| $3x + y$ | 3 |
| $= 3(-1) + 6$ | |
| $= -3 + 6$ | |
| $= 3$ | |

LS = RS

The point $(-1, 6)$ is a solution of the system of equations.

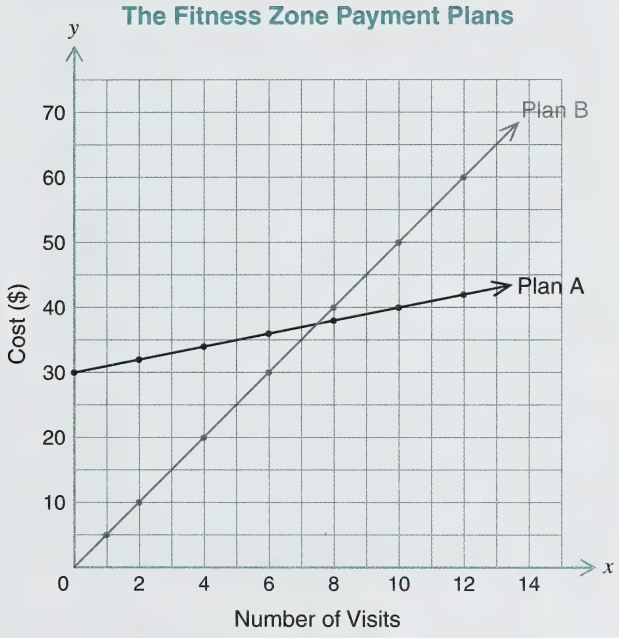
**7. a. Plan A**

| Number of Visits | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Cost ($) | 30 | 32 | 34 | 36 | 38 | 40 | 42 |

**Plan B**

| Number of Visits | 1 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Cost ($) | 5 | 10 | 20 | 30 | 40 | 50 | 60 |

**b.**




**c.** Henry must make a minimum of 8 visits to make Plan A worth while.

**7. Textbook exercises 6, 7, and 8 of "Discussing the Ideas," p. 120**

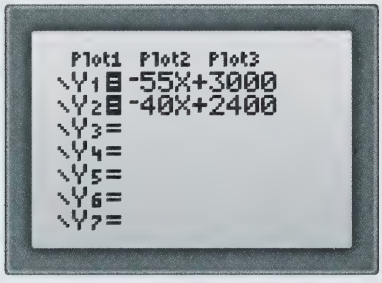
**6.** The $x$-coordinate of the intersection point represents the weekly sales where the two pay schemes give the same income. The $y$-coordinate of the intersection point represents the weekly pay where both schemes give the same income.

**7.** The values Xmin and Ymin are set at zero to reflect no sales. The values of Xmax and Ymax are set so the intersection point shows up away from the edges of the screen. Sometimes trial and error is needed to make a critical point appear on the screen.

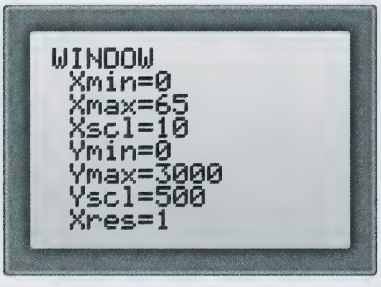
8. The CALC feature is quick and it calculates the coordinates precisely. The TABLE feature may produce a table where values jump over the values corresponding to the intersection point. The TABLE feature is likely to jump over the right values if the coordinates of the intersection point are not whole numbers.

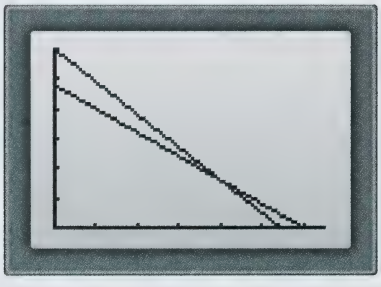
**8. Textbook exercises 5 and 6 of "Exercises: Checking Your Skills," p. 121**

**5. a.** Enter the equations into the equation editor.

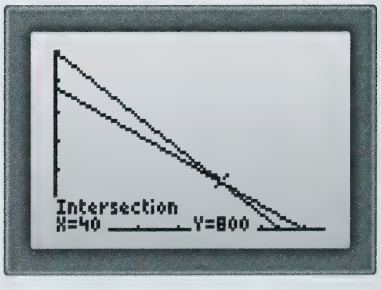



Select suitable window settings, and graph the equations.



**b.** Use the Intersect feature from the CALCULATE menu to determine the point of intersection.



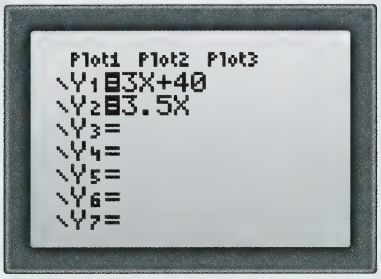

2nd [ CALC ] 5 ENTER ENTER ENTER

The point of intersection is $(40, 800)$.

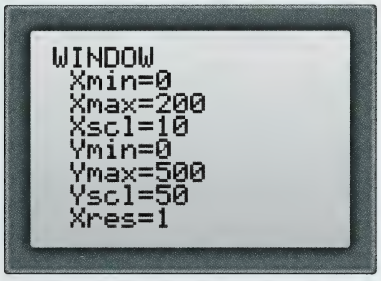
**c.** It will take Tony 40.0 s to overtake Mollie.

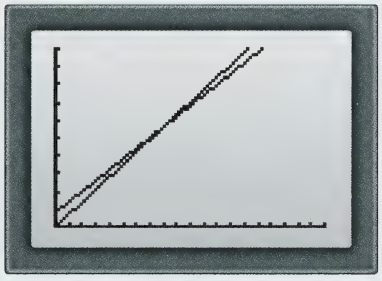
**d.** Tony overtakes Mollie at an altitude of 800 m.

**6. a.** Enter the equations into the equation editor.




Select suitable window settings, and graph the equations.



**b.** Use the Intersect feature from the CALCULATE menu to determine the point of intersection.

2nd [ CALC ] 5 ENTER ENTER ENTER

The point of intersection is $(80, 280)$.

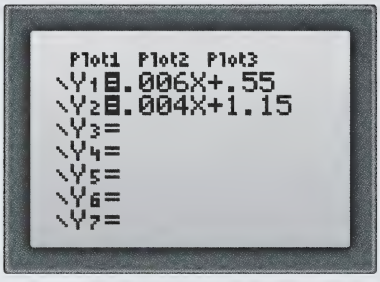



**c.** It takes Lucy 80 s to overtake Julie.

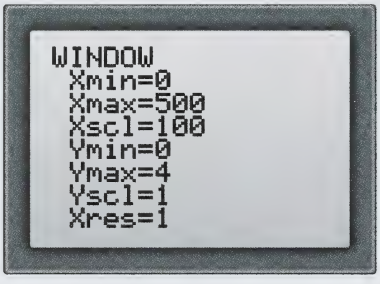
**d.** Lucy overtakes Julie 280 m past the halfway mark.

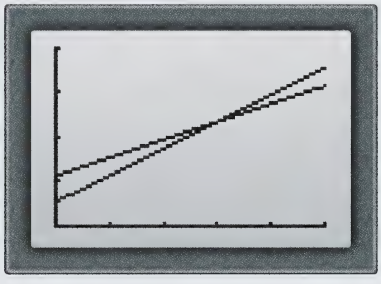
**9. Textbook exercises 8 and 11 of "Exercises: Checking Your Skills, " pp. 122 to 124**

**8. a.** Enter the equations into the equation editor.

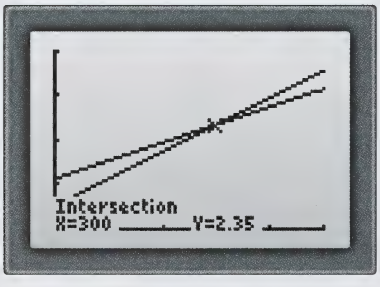



Using the window settings given, the graphs of the system look like the following.
**Remember:** You may need to use trial and error to obtain suitable settings.



Use the Intersect feature from the CALCULATE menu to determine the point of intersection.



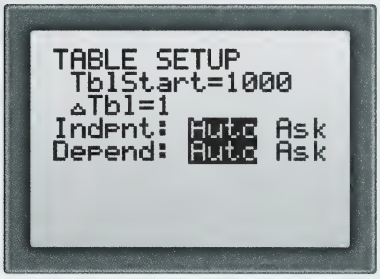

The point $(300, 2.35)$ is a solution of the system.

**b.** It is cheaper to use the energy-saver bulb after 300 h. The cost of using the energy-saver bulb at this point is $2.35.

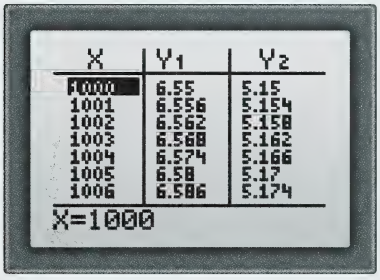
**c.** Determine how much each bulb costs to operate for 1000 h; then determine the difference between the costs.

**Method 1: Using the Table of Values**

**Step 1:** Press [2nd] [ TBLSET ] and set the table of values so it shows the $y$-values for $x = 1000$.




**Step 2:** Press [2nd] to display the table of values.

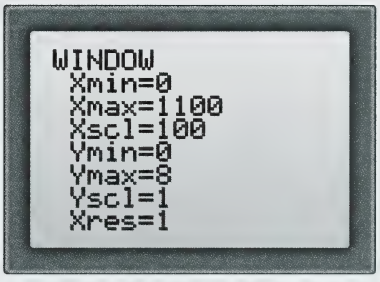



**Step 3:** Determine the difference.

$$6.55 - 5.15 = 1.40$$

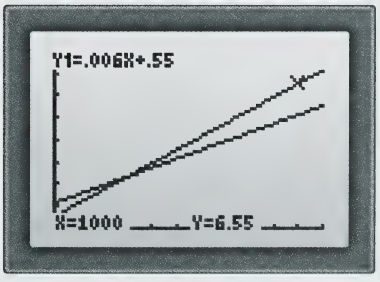
**Method 2: Using the Value Feature**

**Step 1:** Reset the window settings (if necessary) so $x = 1000$ is included in the domain for both graphs.



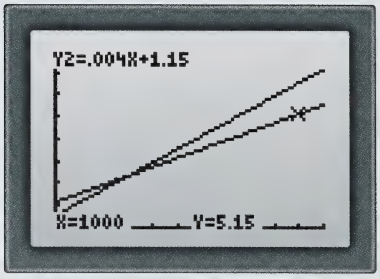

**Step 2:** Determine the cost of using the regular bulb for 1000 h.

[2nd] [ CALC ] [1] (1:value) [1] [0] [0] [0] [ENTER]




The cost of operating the regular bulb for 1000 h is \$6.55.

**Step 3:** Determine the cost of using the energy-saver bulb for 1000 h.




Press ▼ to select the graph of $Y_2$ at $x = 1000$.

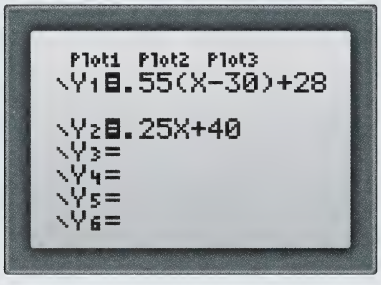
The cost of operating the energy-saver bulb for 1000 h is \$5.15.

**Step 4:** Calculate the difference.

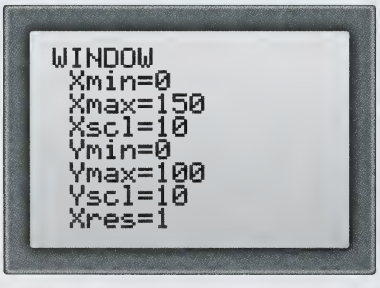
$$6.55 - 5.15 = 1.40$$

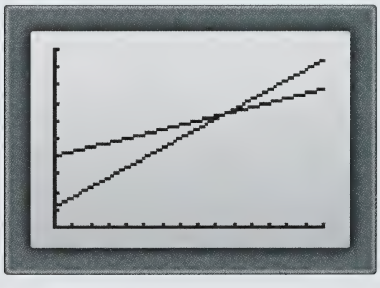
After 1000 h of use, you save \$1.40 using the energy-saver bulb.

**11. a.** Enter the equations into the equation editor.

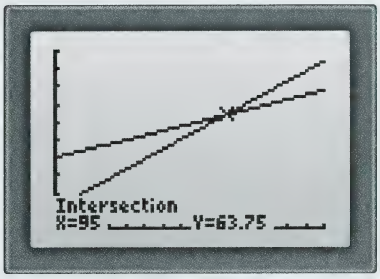



Using the window settings given, the graphs of the system look like the following.



**b.** Use the Intersect feature from the CALCULATE menu to determine the point of intersection.




The point $(95, 63.75)$ is the solution of the system.

Therefore, 95 min of calling time result in the same monthly fee for both plans.

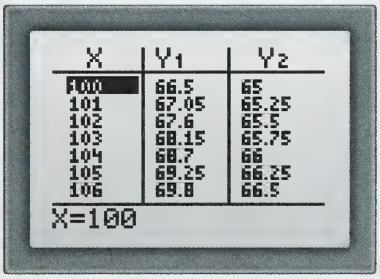
**c.** To the right of the point of intersection, $(95, 63.75)$, the points on the graph of Plan B are less than those on the graph of Plan A. Therefore, Plan B is better if you use the cellular phone for more than 100 min each month.

**Justification**

Find the difference between Plan A and Plan B if you use the phone for 100 min each month.

**Method 1: Using the Table of Values**

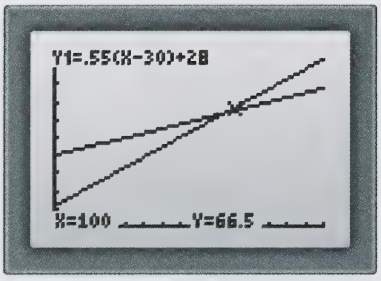


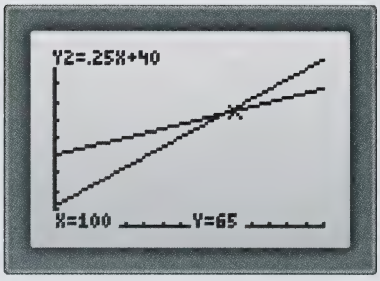



| X | Y1 | Y2 |
|---|---|---|
| 100 | 66.5 | 65 |
| 101 | 67.05 | 65.25 |
| 102 | 67.6 | 65.5 |
| 103 | 68.15 | 65.75 |
| 104 | 68.7 | 66 |
| 105 | 69.25 | 66.25 |
| 106 | 69.8 | 66.5 |

X=100

**Method 2: Using the Value Feature**




By using Plan B, you will save

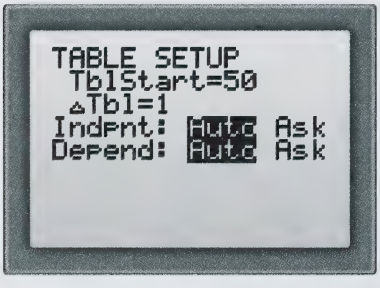
$$\begin{aligned} Y_1 - Y_2 &= \$66.50 - \$65.00 \\ &= \$1.50 \end{aligned}$$

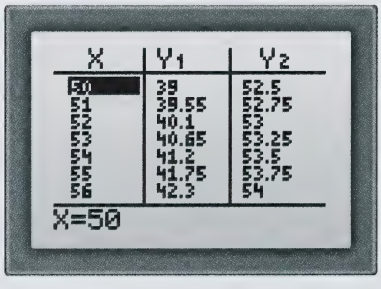
**d.** To the left of the point of intersection, the points on the graph of Plan A are less than those on the graph of Plan B. Therefore, Plan A is better if you use the cellular phone for no more than 50 min each month.

**Justification**

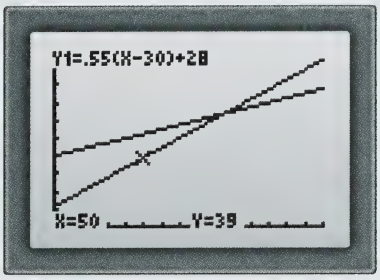
Find the difference between Plan A and Plan B if you use the phone for 50 min each month.

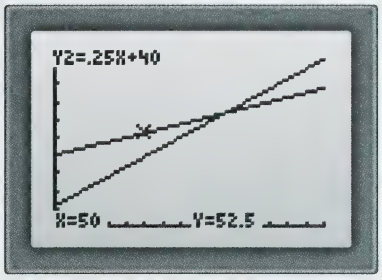
**Method 1: Using the Table of Values**



**Method 1: Using the Value Feature**

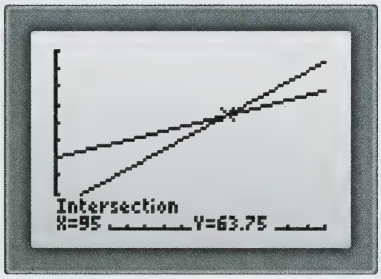



By using Plan A, you will save

$$\begin{aligned} Y_2 - Y_1 &= \$52.50 - \$39.00 \\ &= \$13.50 \end{aligned}$$

**e.** Plan A is more economical (cheaper) to use when the monthly calling time is less than 95 min. The portion of the graph of Plan A to the left of the intersection point is lower than the corresponding portion of the graph of Plan B.




**10. Textbook exercise "Communicating the Ideas," p. 124**

Answers may vary. A sample answer is given.

Graphing and determining the point of intersection to solve a system of equations is much faster and more accurate using a graphing calculator than using paper and pencil. However, it can, at times, be very time-consuming to find suitable window settings.

## Activity 2: Algebraic Modelling

**1. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 129**

**1.** Let $x$ represent the sale price (in dollars) of a shirt and $y$ represent the sale price (in dollars) of a sweater.

**Statement 1:** Two shirts and 4 sweaters cost \$98.

Reword: Twice the cost of a shirt plus 4 times the cost of a sweater is \$98.

Equation 1: $2x + 4y = 98$

**Statement 2:** One shirt and 3 sweaters cost \$69.

Reword: The cost of 1 shirt plus 3 times the cost of a sweater is \$69.

Equation 2: $x + 3y = 69$

The system of linear equations that could be used to determine each unknown is

$2x + 4y = 98$ (1)
$x + 3y = 69$ (2)

**2.** Let $x$ represent the sale price (in dollars) of a CD and $y$ represent the sale price (in dollars) of a tape.

**Statement 1:** Three CDs and 2 tapes cost \$72.

Reword: The cost of 3 CDs plus the cost of 2 tapes is \$72.

Equation 1: $3x + 2y = 72$

**Statement 2:** One CD and 3 tapes cost \$52.

Reword: The cost of 1 CD plus 3 times the cost of a tape is \$52.

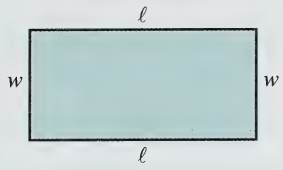
Equation 2: $x + 3y = 52$

The system of linear equations that could be used to determine each unknown is

$3x + 2y = 72$ (1)
$x + 3y = 52$ (2)

**2.** Let $\ell$ represent the length (in cm) of the table and $w$ represent the width (in cm).

Draw a diagram, labelling all sides.




The perimeter is equal to 853.0 cm.

$\therefore\ 2\ell + 2w = 853.0$

The difference between the length and width is 121.5 cm.

$\therefore\ \ell - w = 121.5$

The equations showing the relationship between length, $\ell$, and width, $w$, of a regulation table for table tennis are

$2\ell + 2w = 853.0$ (1)
$\ell - w = 121.5$ (2)

**3. Textbook exercise 2 of "Practise Your Prior Skills," p. 125**

**2.** To calculate the interest earned, substitute the given values for the **p**rincipal, interest **r**ate, and **t**ime (in years) into the formula $I = Prt$ and solve for $I$.

$P = 15\,000,\ r = 0.08,$ and $t = 1$

$\therefore\ I = Prt$
$= (15\,000)(0.08)(1)$
$= 1200$

The interest earned is $1200.

## Activity 2 (continued)

**4. Textbook exercises 6 and 11 of "Exercises: Checking Your Skills," pp. 129 and 130**

**6.** Let $x$ be the amount (in dollars) invested at 7% and $y$ be the amount (in dollars) invested at 10%.

The given information can be represented as follows.

| | Principal ($) | Interest Rate | Time (years) | Interest Earned ($) |
|---|---|---|---|---|
| First Investment | $x$ | 0.07 | 1 | $0.07x$ |
| Second Investment | $y$ | 0.10 | 1 | $0.10y$ |
| Total | 500 | | | 44 |

The equations to determine how much Jennifer invested at each rate are

$x + y = 500$ (1)
$0.07x + 0.10y = 44$ (2)

**11.** Let $x$ be the amount (in dollars) invested at 4% and $y$ be the amount (in dollars) invested at 6%.

The given information can be represented as follows.

| | Principal ($) | Interest Rate | Time (years) | Interest Earned ($) |
|---|---|---|---|---|
| First Investment | $x$ | 0.04 | 1 | $0.04x$ |
| Second Investment | $y$ | 0.06 | 1 | $0.06y$ |
| Total | 1250 | | | |

The equations to determine how much Mona invested at each rate are

$x + y = 1250$ (1)
$0.04x = 0.06y$ (2)

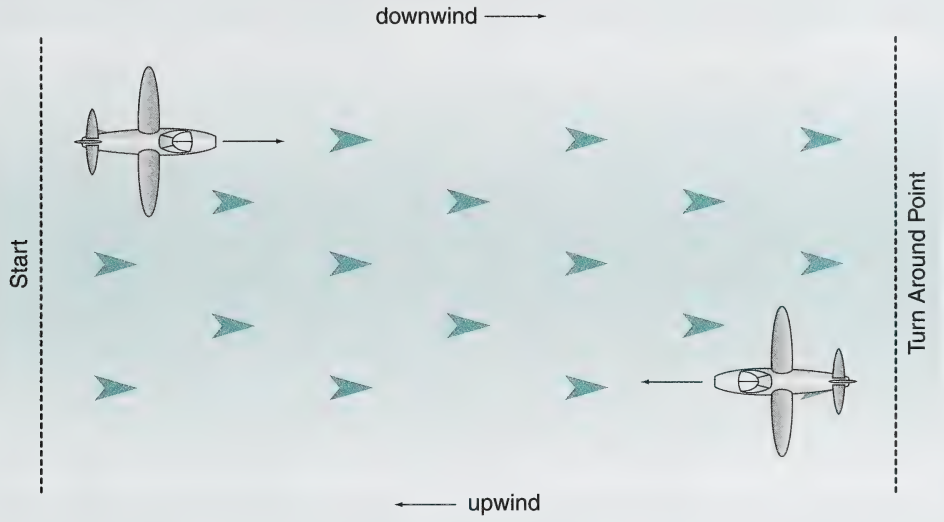
**5. Textbook exercise 1 of "Practise Your Prior Skills," p. 125**

**1.** To determine the average speed, you can use the formula $v = \frac{d}{t}$, where $v$ is the average speed, $d$ is the distance travelled, and $t$ is the elapsed time.

Since $d = vt$, you can rearrange the formula to get $v = \frac{d}{t}$ by dividing both sides by $t$.

**6. Textbook exercises 8 and 10 of "Exercises: Checking Your Skills," p. 130**

**8.**




Let $v$ be the speed (in km/h) of the plane in still air and $d$ be the distance travelled (in km).

| Direction | Speed (km/h) | Time (h) | Distance (km) |
|---|---|---|---|
| Downwind | $v+20$ | 2 | $d$ |
| Upwind | $v-20$ | 3 | $d$ |

Recall that distance = speed × time.

$\therefore d = (v+20)2$ ← downwind $\qquad$ $\therefore d = (v-20)3$ ← upwind

$= 2v+40$ $\qquad$ $= 3v-60$

The equations to determine the speed of the plane when there is no wind and the distance travelled are

$d = 2v+40$ (1)

$d = 3v-60$ (2)

## Activity 2 (continued)

**10.** Let $v$ be the speed (in km/h) of the canoe in still water and $d$ be the distance travelled (in km).

| Direction | Speed (km/h) | Time (h) | Distance (km) |
|---|---|---|---|
| Downstream | $v+8$ | 4 | $d$ |
| Upstream | $v-8$ | 7 | $d$ |

Recall that distance = speed × time.

$\therefore\ d=(v+8)4$ ← downstream
$\quad\ =4v+32$

$\therefore\ d=(v-8)7$ ← upstream
$\quad\ =7v-56$

The equations to determine the speed of the canoe in still water and the distance travelled are

$d=4v+32$ (1)
$d=7v-56$ (2)

**7. Textbook exercise 1 of "Discussing the Ideas," p. 129**

**1.**

| Operation | Sample Cue Words |
|---|---|
| add | plus, and, greater, more, heavier, … |
| subtract | less, minus, difference, lighter, smaller, … |
| multiply | times, product, by, twice, … |
| divide | fraction, quotient, part of, … |

**8. Textbook exercise "Communicating the Ideas," p. 130**

Answers may vary. A sample answer using exercise 5 is given.

### Using Two Unknown Quantities

Let $x$ be the number of shares of World Oil purchased and $y$ be the number of shares of Zinco Mines purchased.

Organize the information in a table.

| Company | Number of Shares Purchased | Share Price ($) | Amount Invested ($) |
|---|---|---|---|
| World Oil | $x$ | 7.50 | $7.50x$ |
| Zinco Mines | $y$ | 3.25 | $3.25y$ |
| Total | 450 | | 2100 |

The equations in two variables representing the problem are

$$x + y = 450 \quad (1)$$
$$7.50x + 3.25y = 2100 \quad (2)$$

### Using One Unknown Quantity

**Method 1**

Let $x$ be the number of shares of World Oil purchased. Therefore, $450 - x$ is the number of shares of Zinco Mines purchased.

Organize the information in a table.

| Company | Number of Shares Purchased | Share Price ($) | Amount Invested ($) |
|---|---|---|---|
| World Oil | $x$ | 7.50 | $7.50x$ |
| Zinco Mines | $450 - x$ | 3.25 | $3.25(450 - x)$ |
| Total | 450 | | 2100 |

The equation in one variable representing the problem is $7.50x + 3.25(450 - x) = 2100$.

## Activity 2 (continued)

**Method 2**

Let $x$ be the number of shares of Zinco Mines purchased. Therefore, $(450-x)$ is the number of shares of World Oil purchased.

Organize the information of the problem in a table.

| Company | Number of Shares Purchased | Share Price ($) | Amount Invested ($) |
|---|---|---|---|
| World Oil | $450-x$ | 7.50 | $7.50(450-x)$ |
| Zinco Mines | $x$ | 3.25 | $3.25x$ |
| Total | 450 | | 2100 |

The equation in one variable representing the problem is $7.50(450-x)+3.25x=2100$.

**9. Textbook exercise 3 of "Discussing the Ideas," p. 129**

**3.** Translating a word problem using two variables seems to be more natural and straight forward. The resulting equations are also simpler than the one equation obtained using only one variable.

## Activity 3: Soving Linear Systems by Substitution

**1. Textbook exercises 1.a., 1.c., and 2.c. of "Practise Your Prior Skills," p. 131**

**1. a.** $2(x+y)-3x+5y=2x+2y-3x+5y$

$=2x-3x+2y+5y$

$=-x+7y$ or $7y-x$

**c.** $4x-5(x-3y)+3(x-y)=4x-5x+15y+3x-3y$

$=4x-5x+3x+15y-3y$

$=2x+12y$

**2. c.** $\frac{1}{4}x+\frac{1}{2}y=34$

$x+2y=136$ ← Multiply both sides by the lowest common multiple (LCM), 4.

**2. Textbook exercises 3.b., 3.c., and 4.c. of "Practise Your Prior Skills," p. 131**

**3. b.** $1-2(x-4)=11$

$1-2x+8=11$ ← Expand by removing the brackets.

$-2x+9=11$ ← Collect like terms.

$-2x=2$ ← Subtract 9 from both sides.

$x=-1$ ← Divide both sides by $-2$.

**c.** $0.04x=2-3(2+0.2x)$

$0.04x=2-6-0.6x$ ← Expand.

$0.04x=-4-0.6x$ ← Collect like terms.

$4x=-400-60x$ ← Multiply both sides by 100 to remove the decimal values.

$64x=-400$ ← Add $60x$ to both sides.

$x=-6.25$ ← Divide both sides by 64.

**4. c.** $-2x+3y=9$

$3y=2x+9$ ← Add $2x$ to both sides.

$y=\frac{2}{3}x+3$ ← Divide both sides by 3.

**3. a.** $3x+5y=5$

$3x+5(-2)=5$ ← Substitute $-2$ for $y$.

$3x-10=5$ ← Simplify.

$3x=15$ ← Add 10 to both sides.

$x=5$ ← Divide both sides by 3.

**b.** $y-12=7x-2$

$-2-12=7x-2$ ← Substitute $-2$ for $y$.

$-14=7x-2$ ← Simplify.

$-12=7x$ ← Add 2 to both sides.

$\frac{-12}{7}=x$ ← Divide both sides by 7.

$x=-\frac{12}{7}$

## Activity 3 (continued)

**4. Textbook exercises 2.a., 4, and 5 of "Exercises: Checking Your Skills," pp. 136 and 137**

**2. a.** $y = 105$ (1)

$-7x + y = 42$ (2)

Substitute 105 for $y$ into equation (2).

$$-7x + y = 42$$
$$-7x + 105 = 42$$
$$-7x = -63 \quad \leftarrow \text{Subtract 105 from both sides.}$$
$$x = 9 \quad \leftarrow \text{Divide both sides by } -7.$$

Check if $x = 9$ and $y = 105$, or $(9, 105)$, is a solution.

| LS | RS |
|---|---|
| $y$ | 105 |
| $= 105$ | |
| LS = | RS |

| LS | RS |
|---|---|
| $-7x + y$ | 42 |
| $= -7(9) + 105$ | |
| $= -63 + 105$ | |
| $= 42$ | |
| LS = | RS |

The solution is $(9, 105)$.

**4.** $d = 90$ (1)

$d = -840t + 5430$ (2)

Substitute 90 for $d$ into equation (2).

$$\begin{aligned} d &= -840t + 5430 \\ 90 &= -840t + 5420 \\ 840t + 90 &= 5430 && \leftarrow \text{Add } 840t \text{ to both sides.} \\ 840t &= 5340 && \leftarrow \text{Subtract 90 from both sides.} \\ t &\doteq 6.357 && \leftarrow \text{Divide both sides by 840.} \end{aligned}$$

Check if $t \doteq 6.357$ and $d = 90$, or $(6.357, 90)$, is a solution.

| LS | RS |
|---|---|
| $d$ | 90 |
| $= 90$ | |
| LS = | RS |

| LS | RS |
|---|---|
| $d$ | $-840t + 5430$ |
| $= 90$ | $\doteq -840(6.357) + 5430$ |
| | $\doteq -5340 + 5430$ |
| | $\doteq 90$ |
| LS = | RS |

The plane has been cruising for approximately 6.36 h when it starts its descent.

**5.** Plan 1: $C = 60$ (1)

Plan 2: $C = 0.25d + 20$ (2)

**a.** Cost of a one-day, 200-km trip.

Plan 1: $C = 60$

The cost is \$60.

For Plan 1, distance is irrelevant.

Plan 2: $C = 0.25d + 20$

$= 0.25(200) + 20$

$= 50 + 20$

$= 70$

The cost is \$70.

Plan 1 is better for the occasion. It is \$10 cheaper.

## Activity 3 (continued)

**b.** Substitute 60 for $C$ into equation (2).

$$C = 0.25d + 20$$
$$60 = 0.25d + 20$$
$$40 = 0.25d \quad \longleftarrow \text{Subtract 20 from both sides.}$$
$$160 = d \quad \longleftarrow \text{Divide both sides by 0.25.}$$

Check if $d = 160$ and $C = 60$, or $(160, 60)$, is a solution.

| LS | RS |
|---|---|
| $C$ | 60 |
| $= 60$ | |

LS = RS

| LS | RS |
|---|---|
| $C$ | $0.25d + 20$ |
| $= 60$ | $= 0.25(160) + 120$ |
| | $= 40 + 20$ |
| | $= 60$ |

LS = RS

The costs for each plan would be the same if you drove 160 km. This rental cost is \$60.

**5. Textbook exercise 3 of "Discussing the Ideas," p. 136**

**3.** The equation $f = s + 60$ was chosen because $f$ is already isolated in this equation.

$$f + s = 300$$
$$f + 120 = 300 \quad \longleftarrow \text{Substitute 120 for } s.$$
$$f = 180 \quad \longleftarrow \text{Subtract 120 from both sides.}$$

The value of $f$ in the solution is still the same.

**6. Textbook exercises 2.c., 6, and 7 of "Exercises: Checking Your Skills," pp. 136 and 137**

**2. c.** $y = 3x$ (1)

$x - 2y = 10$ (2)

Substitute $3x$ for $y$ into equation (2).

$$\begin{aligned} x - 2y &= 10 \\ x - 2(3x) &= 10 \\ x - 6x &= 10 \quad \leftarrow \text{Simplify.} \\ -5x &= 10 \quad \leftarrow \text{Simplify.} \\ x &= -2 \quad \leftarrow \text{Divide both sides by } -5. \end{aligned}$$

Substitute $-2$ for $x$ into equation (1).

$$\begin{aligned} y &= 3x \\ &= 3(-2) \\ &= -6 \end{aligned}$$

Check if $x = -2$ and $y = -6$, or $(-2, -6)$, is a solution.

| LS | RS |
|---|---|
| $y$ | $3x$ |
| $= -6$ | $= 3(-2)$ |
| | $= -6$ |
| LS = | RS |

| LS | RS |
|---|---|
| $x - 2y$ | 10 |
| $= -2 - 2(-6)$ | |
| $= -2 + 12$ | |
| $= 10$ | |
| LS = | RS |

The solution is $(-2, -6)$.

## Activity 3 (continued)

**6. a.** $C = 20h$ (1)

$C = 16h + 12$ (2)

Substitute $20h$ for $C$ into equation (2).

$$\begin{aligned} C &= 16h + 12 \\ 20h &= 16h + 12 \\ 4h &= 12 \quad \longleftarrow \text{Subtract } 16h \text{ from both sides.} \\ h &= 3 \quad \longleftarrow \text{Divide both sides by 4.} \end{aligned}$$

Substitute 3 for $h$ into equation (1).

$$\begin{aligned} C &= 20(3) \\ &= 60 \end{aligned}$$

Check if $h = 3$ and $C = 60$, or $(3, 60)$, is a solution.

| LS | RS |
|---|---|
| $C$ | $20h$ |
| $= 60$ | $= 20(3)$ |
| | $= 60$ |
| LS | = RS |

| LS | RS |
|---|---|
| $C$ | $16h + 12$ |
| $= 60$ | $= 16(3) + 12$ |
| | $= 48 + 12$ |
| | $= 60$ |
| LS | = RS |

A trail ride of 3 h would result in the same costs for each stable.

**b.** Substitute 2 for $h$ into both equations.

$$\begin{aligned} C &= 20h \quad (1) \\ &= 20(2) \\ &= 40 \end{aligned}$$

$$\begin{aligned} C &= 16h + 12 \quad (2) \\ &= 16(2) + 12 \\ &= 32 + 12 \\ &= 44 \end{aligned}$$

If you go riding for 2 h, Pyramid Stables is less expensive. Assuming other factors are the same, you should choose Pyramid Stables.

**7.** Supply: $G = 5000p - 5000$ (1)

Demand: $G = -4000p + 26\,000$ (2)

Equation (1) shows that the greater the price of grapes, the more the farmers will produce. It also shows that if the price of grapes were \$1/kg or less, farmers would not supply grapes at all.

Equation (2) shows that if grapes were free, consumers would want 26 000 kg. It also shows that as the price of grapes rises, consumers will buy less grapes. In fact, for every \$1/kg that the price of grapes increases, the demand for grapes will decrease by 4000 kg.

Substitute $5000p - 5000$ for $G$ into equation (2).

$$G = -4000p + 26\,000$$
$$5000p - 5000 = -4000p + 26\,000$$
$$9000p - 5000 = 26\,000 \quad \leftarrow \text{Add } 4000p \text{ to both sides.}$$
$$9000p = 31\,000 \quad \leftarrow \text{Add } 5000 \text{ to both sides.}$$
$$p \doteq 3.4444 \quad \leftarrow \text{Divide both sides by } 9000.$$

Substitute 3.4444 for $p$ into equation (1).

$$G = 5000p - 5000$$
$$\doteq 5000(3.4444) - 5000$$
$$\doteq 12\,222$$

Check if $p \doteq 3.4444$ and $G \doteq 12\,222$, or $(3.4444, 12\,222)$, is a solution.

| LS | RS |
|---|---|
| $G$ | $(9,105).5000p - 5000$ |
| $\doteq 12\,222$ | $\doteq 5000(3.4444) - 5000$ |
| | $\doteq 12\,222$ |

LS = RS

| LS | RS |
|---|---|
| $G$ | $-4000p + 26\,000$ |
| $\doteq 12\,222$ | $\doteq -4000(3.4444) + 26\,000$ |
| | $\doteq -13\,777.6 + 26\,000$ |
| | $\doteq 12\,222$ |

LS = RS

The supply of grapes equals the demand when the price of grapes is \$3.44/kg.

## Activity 3 (continued)

**7. Textbook exercises 3.a., 3.d., 8, 11, 12, and 14 of "Exercises: Checking Your Skills," pp. 137 to 139**

**3. a.** $x+y=5$ (1)
$2x-y=-4$ (2)

Isolate $y$ in equation (1).

$x+y=5$
$y=5-x$ ← Subtract $x$ from both sides.

Substitute $5-x$ for $y$ into equation (2).

$2x-y=-4$
$2x-(5-x)=-4$
$2x-5+x=-4$ ← Expand.
$3x-5=-4$ ← Simplify.
$3x=1$ ← Add 5 to both sides.
$x=\frac{1}{3}$ ← Divide both sides by 3.

Substitute $\frac{1}{3}$ for $x$ into equation (1).

$x+y=5$
$\frac{1}{3}+y=5$
$y=4\frac{2}{3}$ ← Subtract $\frac{1}{3}$ from both sides.

Check if $x=\frac{1}{3}$ and $y=4\frac{2}{3}$, or $\left(\frac{1}{3}, 4\frac{2}{3}\right)$, is a solution.

| LS | RS |
|---|---|
| $x+y$ | $5$ |
| $=\frac{1}{3}+4\frac{2}{3}$ | |
| $=5$ | |

LS = RS

| LS | RS |
|---|---|
| $2x-y$ | $-4$ |
| $=2\left(\frac{1}{3}\right)-4\frac{2}{3}$ | |
| $=\frac{2}{3}-4\frac{2}{3}$ | |
| $=-4$ | |

LS = RS

The solution is $\left(1, 4\frac{2}{3}\right)$.

**d.** $x-y=2$ (1)
$3x+y=-14$ (2)

Isolate $x$ in equation (1).

$x-y=2$
$x=2+y$ ← Add $y$ to both sides.

Substitute $2+y$ for $x$ into equation (2).

$3x+y=-14$
$3(2+y)+y=-14$
$6+3y+y=-14$
$6+4y=-14$
$4y=-20$ ← Subtract 6 from both sides.
$y=-5$ ← Divide both sides by 4.

Substitute $-5$ for $y$ into equation (1).

$x-y=2$
$x-(-5)=2$
$x+5=2$
$x=-3$

Check if $x=-3$ and $y=-5$, or $(-3,-5)$, is a solution.

| LS | RS |
|---|---|
| $x-y$ | 2 |
| $=(-3)-(-5)$ | |
| $=-3+5$ | |
| $=2$ | |

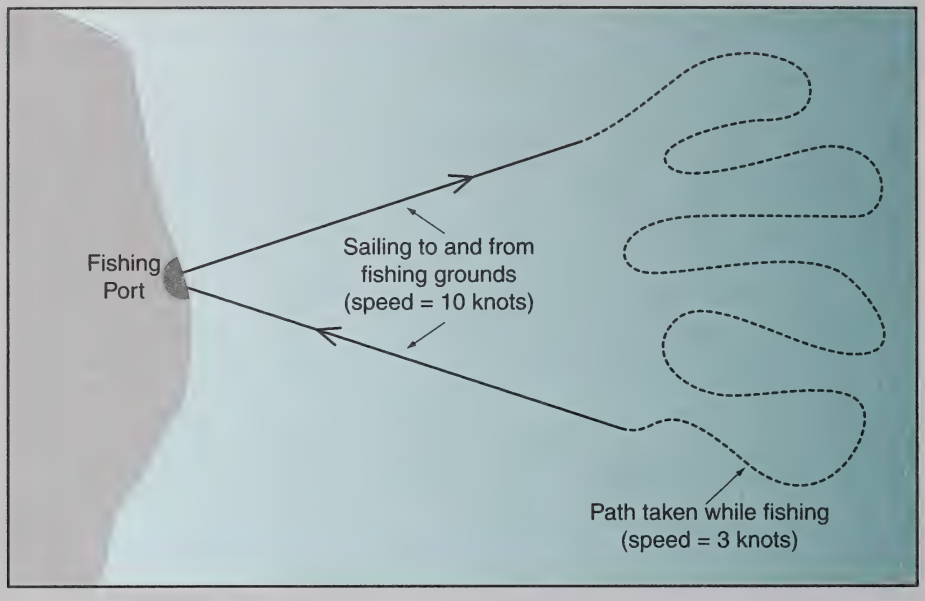
LS = RS

| LS | RS |
|---|---|
| $3x+y$ | $-14$ |
| $=3(-3)+(-5)$ | |
| $=-9-5$ | |
| $=-14$ | |

LS = RS

The solution is $(-3,-5)$.

**8.** The equations can be developed this way.




Let $x$ be the time (in hours) spent sailing and $y$ be the time (in hours) spent fishing.

Organize the information in a table.

| | Speed (knots) | Time (h) | Distance (nautical miles) |
|---|---|---|---|
| Sailing | 10 | $x$ | $10x$ |
| Fishing | 3 | $y$ | $3y$ |
| Total | | 11 | 61 |

$\therefore\ x + y = 11$ (1)

$10x + 3y = 61$ (2)

To answer the questions, you must solve the following system of equations:

$$x+y=11 \quad (1)$$
$$10x+3y=61 \quad (2)$$

Isolate $y$ in equation (1).

$$x+y=11$$
$$y=11-x$$

Substitute $11-x$ for $y$ into equation (2).

$$\begin{aligned} 10x+3y &= 61 \\ 10x+3(11-x) &= 61 \\ 10x+33-3x &= 61 \\ 7x+33 &= 61 \\ 7x &= 28 \\ x &= 4 \end{aligned}$$

Substitute 4 for $x$ into equation (1).

$$x+y=11$$
$$4+y=11$$
$$y=7$$

Check if $x=4$ and $y=7$, or $(4,7)$, is a solution of the system.

| LS | RS |
|---|---|
| $x+y$ | 11 |
| $=4+7$ | |
| $=11$ | |

LS = RS

| LS | RS |
|---|---|
| $10x+3y$ | 61 |
| $=10(4)+3(7)$ | |
| $=40+21$ | |
| $=61$ | |

LS = RS

The solution is $(4,7)$.

**a.** The time spent sailing to and from the fishing grounds was 4 h.

**b.** The time spent fishing was 7 h.

Check if answer fits with information given in the problem.

The time away is 4 h sailing and 7 h fishing. The total time away is 11 h, which was given as the total time away.

The total distance travelled is the sum of speed × time for each portion (sailing and fishing) of the trip.

$$(10 \text{ nautical miles/h} \times 4 \text{ h}) + (3 \text{ nautical miles/h} \times 7 \text{ h})$$
$$= 40 \text{ nautical miles} + 21 \text{ nautical miles}$$
$$= 61 \text{ nautical miles}$$

This distance agrees with the value given in the problem.

**11. a.**

| | Number Sold | Amount of Meat |
|---|---|---|
| Quarter-pound burgers | $x$ | $\frac{1}{4}x$ |
| Half-pound burgers | $y$ | $\frac{1}{2}y$ |
| Total | | 1250 |

**b.** $x = 3y$ (1)

$\frac{1}{4}x + \frac{1}{2}y = 1250$ (2)

**c.** Substitute $3y$ for $x$ into equation (2).

$$\frac{1}{4}x + \frac{1}{2}y = 1250$$
$$\frac{1}{4}(3y) + \frac{1}{2}y = 1250$$
$$\frac{3}{4}y + \frac{1}{2}y = 1250$$
$$\frac{5}{4}y = 1250 \quad \leftarrow \text{Simplify.}$$
$$y = 1000 \quad \leftarrow \text{Multiply both sides by } \tfrac{4}{5}.$$

Substitute 1000 for $y$ into equation (1).

$$\begin{aligned} x &= 3y \\ &= 3(1000) \\ &= 3000 \end{aligned}$$

Check if $x = 3000$ and $y = 1000$, or $(3000, 1000)$, is a solution.

| LS | RS |
|---|---|
| $x$ | $3y$ |
| $= 3000$ | $= 3(1000)$ |
| | $= 3000$ |

LS = RS

| LS | RS |
|---|---|
| $\frac{1}{4}x + \frac{1}{2}y$ | 1250 |
| $= \frac{1}{4}(3000) + \frac{1}{2}(1000)$ | |
| $= 750 + 500$ | |
| $= 1250$ | |

LS = RS

Therefore, 3000 quarter-pound burgers and 1000 half-pound burgers are made and sold in a typical month.

**d.** A total of 4000 burgers are sold each month.

**12.** Let $x$ be the number of candy bars purchased and $y$ be the number of cans of pop purchased.

| | Number Purchased | Cost ($) |
|---|---|---|
| Candy bars | $x$ | $0.45x$ |
| Cans of pop | $y$ | $0.48y$ |
| Total | | 11.40 |

The system of equations is as follows:

$$0.45x + 0.48y = 11.40 \quad (1)$$
$$x = 4y \quad (2)$$

## Activity 3 (continued)

Substitute $4y$ for $x$ into equation (1).

$$\begin{aligned} 0.45x + 0.48y &= 11.40 \\ 0.45(4y) + 0.48y &= 11.40 \\ 1.80y + 0.48y &= 11.40 \\ 2.28y &= 11.40 \\ y &= 5 \end{aligned}$$

Substitute 5 for $y$ into equation (2).

$$\begin{aligned} x &= 4y \\ &= 4(5) \\ &= 20 \end{aligned}$$

Check if $x = 20$ and $y = 5$, or $(20, 5)$, is a solution.

| LS | RS |
|---|---|
| $0.45x + 0.48y$ | $11.40$ |
| $= 0.45(20) + 0.48(5)$ | |
| $= 9.00 + 2.40$ | |
| $= 11.40$ | |

LS = RS

| LS | RS |
|---|---|
| $x$ | $4y$ |
| $= 20$ | $= 4(5)$ |
| | $= 20$ |

LS = RS

Liz bought 20 candy bars and 5 cans of pop.

**14.** Answers will vary. A sample answer is given.

You can calculate the total spent when buying

- 1 can of pop and 4 candy bars
- 2 cans of pop and 8 candy bars
- 3 cans of pop and 12 candy bars

⋮

Use trial and error until the total amount spent equals $11.40. You can do this manually or by using a spreadsheet.

Using a spreadsheet, set up a table and enter these formulas.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Cans of Pop | | Candy Bars | | Total Amount Spent ($) |
| 2 | Number Purchased | Cost ($) | Number Purchased | Cost ($) | |
| 3 | 1 | =A3*0.48 | =A3*4 | =C3*0.45 | =B3+D3 |
| 4 | =A3+1 | =A4*0.48 | =A4*4 | =C4*0.45 | =B4+D4 |
| 5 | =A4+1 | =A5*0.48 | =A5*4 | =C5*0.45 | =B5+D5 |
| 6 | =A5+1 | =A6*0.48 | =A6*4 | =C6*0.45 | =B6+D6 |
| 7 | =A6+1 | =A7*0.48 | =A7*4 | =C7*0.45 | =B7+D7 |
| 8 | =A7+1 | =A8*0.48 | =A8*4 | =C8*0.45 | =B8+D8 |
| 9 | =A8+1 | =A9*0.48 | =A9*4 | =C9*0.45 | =B9+D9 |

The values of each cell are as follows.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Cans of Pop | | Candy Bars | | Total Amount Spent ($) |
| 2 | Number Purchased | Cost ($) | Number Purchased | Cost ($) | |
| 3 | 1 | $0.48 | 4 | $1.80 | $2.28 |
| 4 | 2 | $0.96 | 8 | $3.60 | $4.56 |
| 5 | 3 | $1.44 | 12 | $5.40 | $6.84 |
| 6 | 4 | $1.92 | 16 | $7.20 | $9.12 |
| 7 | 5 | $2.40 | 20 | $9.00 | $11.40 |
| 8 | 6 | $2.88 | 24 | $10.80 | $13.68 |
| 9 | 7 | $3.36 | 28 | $12.60 | $15.96 |

## Activity 3 (continued)

**8. Textbook exercise "Communicating the Ideas," p. 139**

Answers may vary. A sample answer is given.

### Solving a System of Linear Equations by Substitution

**Step 1:** Isolate one of the variables in one of the equations.

**Step 2:** Substitute the equivalent expression for the isolated variable into the other equation.

**Step 3:** Solve the resulting equation for the unknown variable.

**Step 4:** Substitute the value of the variable found in Step 3 into one of the original equations.

**Step 5:** Solve the equation for the other unknown variable (the variable that was isolated in Step 1).

**Step 6:** Check if the values are a solution to the system. Make sure the solution satisfies both equations.

**Step 7:** State the solution.

**Verification**

Use Steps 1 to 7 to answer exercise 3.d. on page 137 of the textbook.

$x - y = 2$ (1)
$3x + y = -14$ (2)

**Step 1: Isolate one of the variables in one of the equations.**

$$x - y = 2$$
$$x = 2 + y$$

**Step 2: Substitute the equivalent expression for the isolated variable into the other equation.**

$$3x + y = -14$$
$$3(2 + y) + y = -14$$

**Step 3: Solve the resulting equation for the unknown variable.**

$$\begin{aligned} 3(2+y)+y &= -14 \\ 6+3y+y &= -14 \\ 6+4y &= -14 \\ 4y &= -20 \\ y &= -5 \end{aligned}$$

**Step 4: Substitute the value of the variable found in Step 3 into one of the original equations.**

$$\begin{aligned} x-y &= 2 \\ x-(-5) &= 2 \end{aligned}$$

**Step 5: Solve the equation for the other unknown variable.**

$$\begin{aligned} x-(-5) &= 2 \\ x+5 &= 2 \\ x &= -3 \end{aligned}$$

**Step 6: Check if the values are a solution to the system. Substitute $-3$ for $x$ and $-5$ for $y$ into both original equations.**

| LS | RS |
|---|---|
| $x-y$ | 2 |
| $=-3-(-5)$ | |
| $=-3+5$ | |
| $=2$ | |

LS = RS

| LS | RS |
|---|---|
| $3x+y$ | $-14$ |
| $=3(-3)+(-5)$ | |
| $=-9-5$ | |
| $=-14$ | |

LS = RS

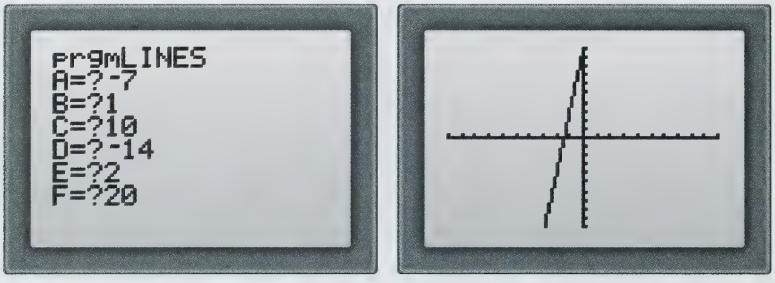
**Step 7: State the solution.**

The solution is $x=-3$ and $y=-5$, or $(-3,-5)$.

## Activity 4: Properties of Linear Systems

**1. Textbook exercise 1 of "Investivation 1: The Number of Solutions of a Linear System," p. 142**

**1.** $-7x+y=10$ (1)
$-14x+2y=20$ (2)




**a.** The graphs of the equations coincide. (They are in the same position.)

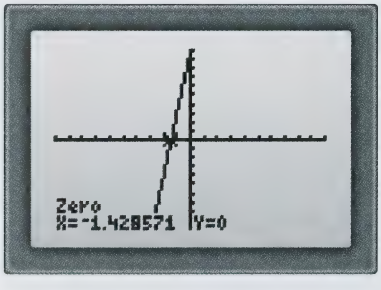
Because the graphs coincide, use only one equation to find the slope of both graphs. Transform equation (1) into slope-intercept form, $y=mx+b$.

$$-7x+y=10$$
$$y=7x+10$$

($m$ = slope, $b$ = $y$-intercept)

The slope, $m$, of both equations is 7.

Use the Zero feature from the CALCULATE menu to find the $x$-intercept. (Refer to Module 2 to refresh your memory as to how to use this feature.)




The $x$-intercept of both graphs is about $-1.43$, and the $y$-intercept is 10 (from the slope-intercept form).

**b.** The two graphs intersect at an infinite number of points because the lines coincide.

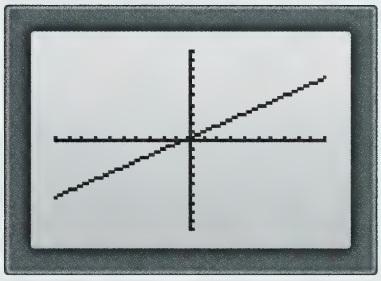
**c.** This system has an infinite number of solutions.

**d.** The equations are alike in that corresponding coefficients form the same ratio 2:1. The final constant terms also form the same ratio. In fact, the second equation is a multiple of the first. The equations are different in that their corresponding coefficients and constant terms are unequal.

**e.** A system in which one equation is a multiple of the other has an infinite number of solutions.

**f.** Answers will vary. The following is another system of equations where one is a multiple of the other.

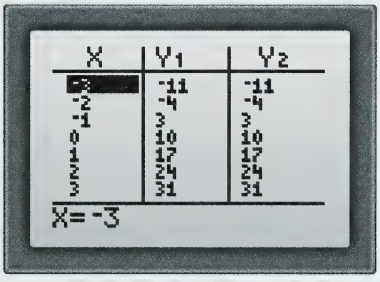
$2x - 3y = -1$ (1)
$6x - 9y = -3$ (2)



The graphs coincide, showing an infinite number of intersection points and an infinite number of solutions.

**2.** Answers will vary. With the system already graphed (using the LINES program), use the table of values to find four points that satisfy the system.




Four points that satisfy the system of equations are $(-3, -11)$, $(-1, 3)$, $(1, 17)$, and $(3, 31)$.

**Note:** You could have also used the Value feature or the Intersect feature, both from the CALCULATE menu, to find points that satisfy the system.

## Activity 4 (continued)

**3. Textbook exercises 2 and 3 of "Investigation 1: The Number of Solutions of a Linear System," pp. 142 and 143**

**2.** $2x-5y=15$ (1)
$2x-5y=-5$ (2)




**a.** The graphs are parallel. To find their slopes and $y$-intercepts, rewrite both equations in slope-intercept form, $y=mx+b$.

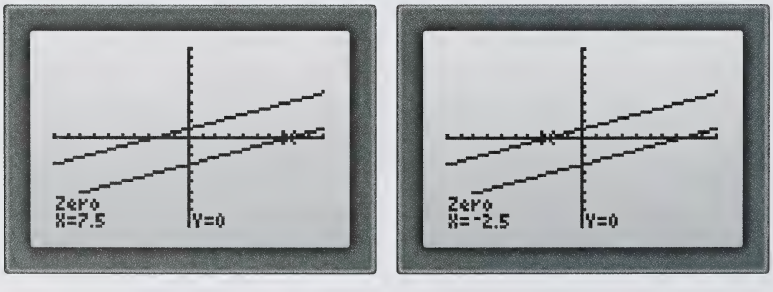
$2x-5y=15$ (1)
$-5y=-2x+15$
$y=\frac{2}{5}x-3$
($m$, $b$)

$2x-5y=-5$ (2)
$-5y=-2x-5$
$y=\frac{2}{5}x+1$
($m$, $b$)

The slope of both equations is $\frac{2}{5}$.

The $y$-intercept of equation (1) is $-3$, and the $y$-intercept of equation (2) is 1.

To find the $x$-intercepts of both equations, use the Zero feature from the CALCULATE menu.




The $x$-intercept of equation (1) is 7.5, and the $x$-intercept of equation (2) is $-2.5$.

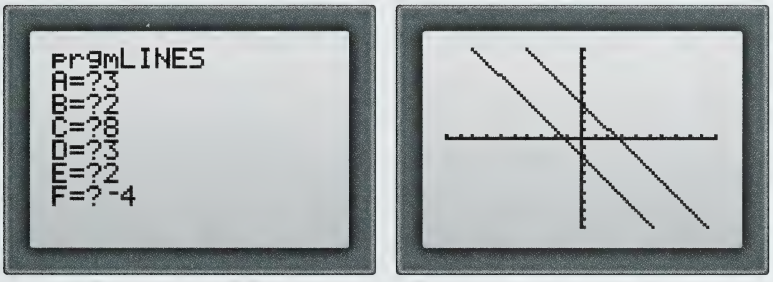
**b.** The lines do not intersect; they are parallel. There is no point of intersection and, therefore, no solution.

**c.** The equations are alike in that corresponding coefficients are equal; so, their slopes are equal. The equations are different in that the constant terms are unequal; so, the $y$-intercepts are different.

**d.** A system will have no solution when the corresponding coefficients in the equation are equal and the constant terms are different. (The graphs are parallel.)

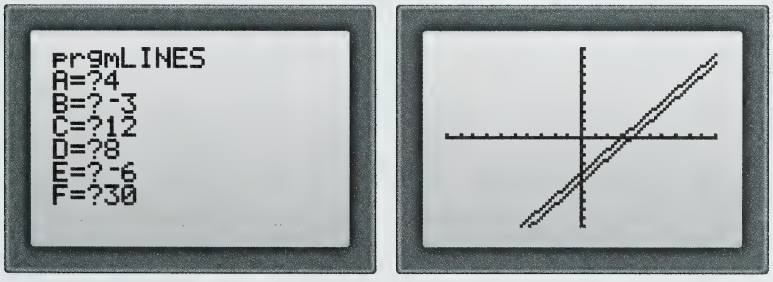
**e.** The corresponding coefficients form a 1 to 1 ratio, and the corresponding constant terms form a 2 to $-1$ ratio. Therefore, the system has no solution.

Test the prediction using the LINES program. **Note:** If the program is still operating, you do not have to restart the program when you want to enter new values for A to F. Simply press CLEAR to clear the graph; then press ENTER to start entering new values for A to F.




The display shows that the lines are parallel. (This is confirmed when an error message appears when using the Intersect feature.) Therefore, the prediction is correct.

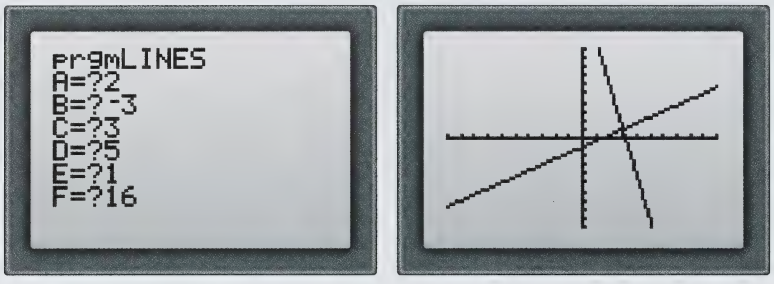
**f.** $4x-3y=12$ (1)
$8x-6y=30$ (2)




The system has no solutions.

**g.** In the equations in 1.f., the corresponding coefficients make ratios of 1 to 2 and the corresponding constants make a ratio of 2 to 5. In general, a system will have no solution when the corresponding coefficients in the equations are equal in ratio and the constant terms are of a different ratio.

**3.** $2x-3y=3$ (1)
$5x+y=16$ (2)




**a.** The graphs intersect.

**b.** The graphs of the two equations intersect at one point.

**c.** Besides being linear equations in the same two variables, the equations have little in common. The coefficients of $x$ are unequal in the equations; the coefficients of $y$ are unequal and form a different ratio than that of the coefficients of $x$; and the constants are unequal and form another unique ratio.

**4. Textbook exercises 3 and 5 of "Exercises: Checking Your Skills," p. 147**

**3. a.** The pairs of lines that would create a system of equations with exactly one solution are A and C or B and C.

**b.** The pair of lines that would create a system with no solution is A and B.

**5. a.** Answers will vary. Sample answers are given.

The following are all the possible linear systems of two equations that have no solution. In each system, the ratios of the corresponding coefficients are the same, but the ratio of the constants are different.

$4x+2y=20$
$6x+3y=5$
2:3 2:3 4:1

$2x+1y=10$
$6x+3y=5$
1:3 1:3 2:1

$5x-15y=-60$
$1x-3y=12$
5:1 5:1 −5:1

**b.** The following are three possible linear systems of two equations that have only one solution. In each system, the ratios of the corresponding coefficients and the ratio of the constants are different.

$$\begin{aligned} 4x+2y&=20\\ 1x-3y&=12 \end{aligned}$$

4:1 2:−3 5:3

$$\begin{aligned} 4x+2y&=20\\ 5x-15y&=-60 \end{aligned}$$

4:5 2:−15 1:−3

$$\begin{aligned} 4x+2y&=20\\ 2x-6y&=24 \end{aligned}$$

2:1 1:−3 5:6

**c.** The following are all the possible linear systems of two equations that have infinitely many solutions. In each system, the ratios are all the same; one equation is simply a multiple of the other.

$$\begin{aligned} 4x+2y&=20\\ 2x+1y&=10 \end{aligned}$$

2:1 2:1 2:1

$$\begin{aligned} 1x-3y&=12\\ 2x-6y&=24 \end{aligned}$$

1:2 1:2 1:2

**5. a.** **No Solution**

$4x+2y=20 \longrightarrow y=-2x+10$

$6x+3y=5 \longrightarrow y=-2x+\frac{5}{3}$

$2x+y=10 \longrightarrow y=-2x+10$

$6x+3y=5 \longrightarrow y=-2x+\frac{5}{3}$

$5x-15y=-60 \longrightarrow y=\frac{1}{3}x+4$

$x-3y=12 \longrightarrow y=\frac{1}{3}x-4$

In each system, the values of $m$ are equal and the values of $b$ are different.

**One Solution**

$2x+y=10 \longrightarrow y=-2x+10$

$x-3y=12 \longrightarrow y=\frac{1}{3}x-4$

$x-3y=12 \longrightarrow y=\frac{1}{3}x-4$

$6x+3y=5 \longrightarrow y=-2x+5$

**Note:** The systems shown here are two additional linear systems that may have appeared in exercise 5.b. on page 147 of the textbook.

In each system, the values of $m$ are different. The values of $b$ are also different in these cases, but they don't have to be.

## Activity 4 (continued)

### Infinitely Many Solutions

$4x+2y=20 \longrightarrow y=-2x+10$

$2x+y=10 \longrightarrow y=-2x+10$

$x-3y=12 \longrightarrow y=\frac{1}{3}x-4$

$2x-6y=24 \longrightarrow y=\frac{1}{3}x-4$

In each system, the values of $m$ are equal and the values of $b$ are equal.

**b.** For inconsistant systems, the values of $m$ must be equal and the values of $b$ must be different.

**6. Textbook exercises 1, 2.b., 2.c., 4.a., and 4.b. of "Exercises: Checking Your Skills," p. 147**

**1. a.** $x-y=5$ (1)

$3x+4y=-6$ (2)

Check if $(2,-3)$ is a solution.

| LS | RS |
|---|---|
| $x-y$ | 5 |
| $=2-(-3)$ | |
| $=2+3$ | |
| $=5$ | |

LS = RS

| LS | RS |
|---|---|
| $3x+4y$ | $-6$ |
| $=3(2)+4(-3)$ | |
| $=6+(-12)$ | |
| $=-6$ | |

LS = RS

The ordered pair $(2,-3)$ is a solution of the system.

**b.** $2x+y=7$ (1)
$x-3y=10$ (2)

Check if $(2,-3)$ is a solution.

| LS | RS |
|---|---|
| $2x+y$ | 7 |
| $=2(2)+(-3)$ | |
| $=4-3$ | |
| $=1$ | |

LS $\neq$ RS

| LS | RS |
|---|---|
| $x-3y$ | 10 |
| $=2-3(-3)$ | |
| $=2+9$ | |
| $=11$ | |

LS $\neq$ RS

The ordered pair $(2,-3)$ is not a solution of the system.

**c.** $4x-y=11$ (1)
$-12x+3y=-16$ (2)

Check if $(2,-3)$ is a solution.

| LS | RS |
|---|---|
| $4x-y$ | 11 |
| $=4(2)-(-3)$ | |
| $=8+3$ | |
| $=11$ | |

LS = RS

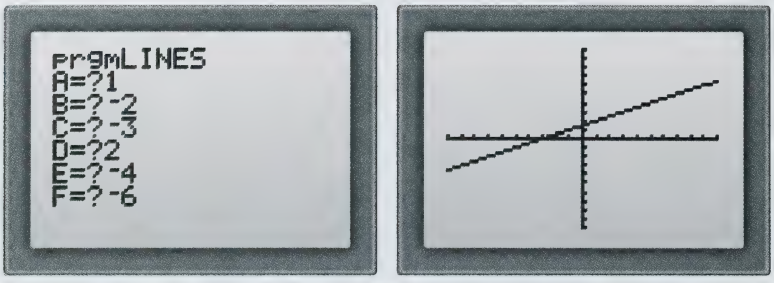
| LS | RS |
|---|---|
| $-12x+3y$ | $-16$ |
| $=-12(2)+3(-3)$ | |
| $=-24-9$ | |
| $=-31$ | |

LS $\neq$ RS

The ordered pair $(2,-3)$ is not a solution of the system.

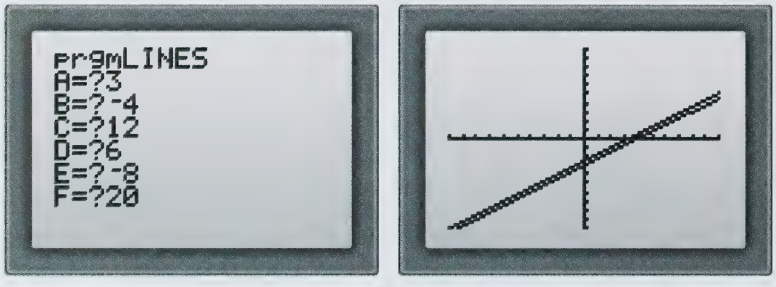
The ordered pair $(2,-3)$ is a solution of system 1.a. only.

**2. b.** The systems can be graphed using the LINES program.




The system consists of two lines coinciding. Therefore, the system has an infinite number of solutions.

**c.**




The system consists of two parallel lines. Therefore, the system has no solution.

**4. a.** $x+2y=6$

$x+2y=2$

The corresponding coefficients are equal, and the constants are not equal. Therefore, the system has no solution.

**b.** $3x+5y=9$

$6x+10y=18$

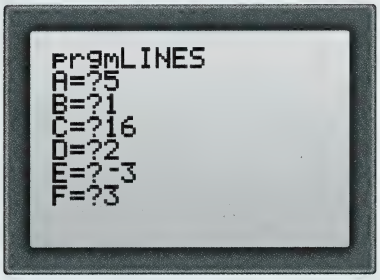
1:2 1:2 1:2

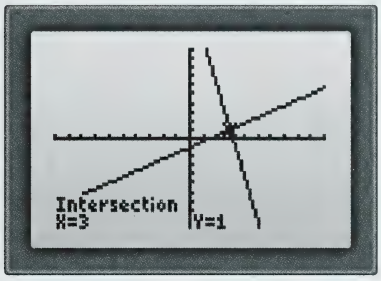
The ratios of the corresponding coefficients and the constants are the same. Therefore, the system has an infinite number of solutions.

**7. Textbook exercises 1 to 6 of "Investigation 2: Properties of Linear Systems," p. 144**

**1.** $5x+y=16$ (1)
$2x-3y=3$ (2)

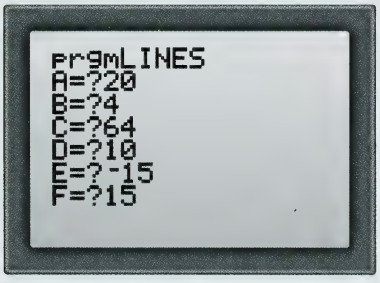
Graph the equations; then use the Intersect feature from the CALCULATE menu to find the solution.

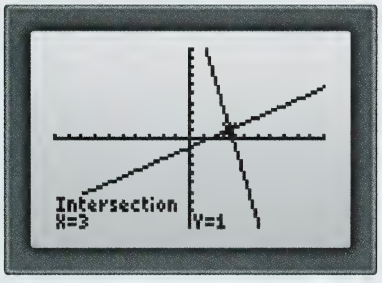



The solution is $(3,1)$.

**2.** $20x+4y=64$ (1)
$10x-15y=15$ (2)



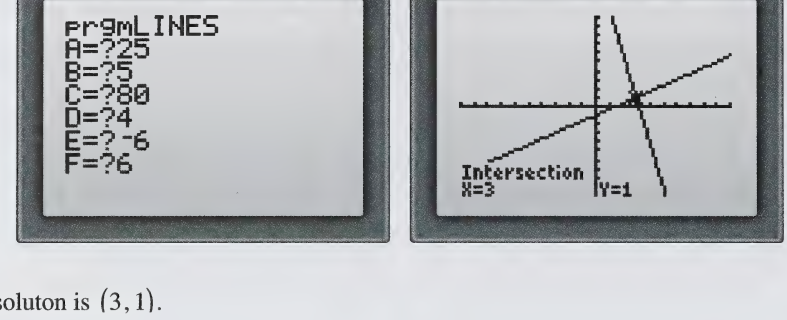




The solution is $(3,1)$.

**3.** The graphs are the same, and the point of intersection occurs at the same point.

**4.** Multiplying an equation by a constant produces another equation with a graph that coincides with the graph of the original equation.

**5.** $25x+5y=80$ ⑤ : 5 × ①
$4x-6y=6$ ⑥ : 2 × ②




The soluton is $(3,1)$.

The graphs are the same even though the equations of the system were multiplied by constant values.

**6.** Multiplying an equation of a linear system by any constant does not change the solution of the system.

**8. Textbook exercises 1 to 7 of "Investigation 3: Adding or Subtracting the Equations of a Linear System," pp. 144 and 145**

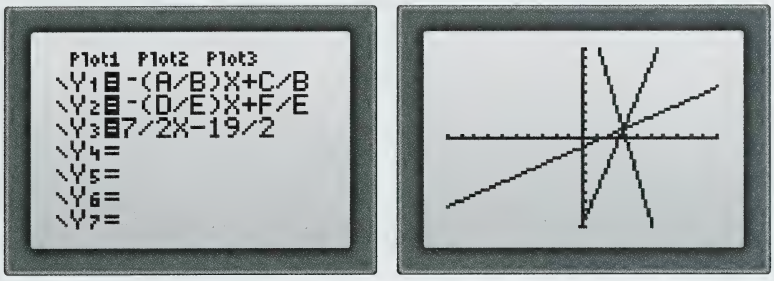
**1.** Add equations ① and ②.

$$\begin{array}{rl} 5x+\ y=16 & ① \\ \underline{2x-3y=\ 3} & ② \\ 7x-2y=19 & \end{array}$$

Rearrange the equation to isolate $y$.

$$\begin{aligned} 7x-2y&=19 \\ -2y&=-7x+19 \\ y&=\frac{7}{2}x-\frac{19}{2} \end{aligned}$$

2. Graph the equation by entering it into the equation editor. Leave $Y_1$ and $Y_2$ as filled in by the LINES program based on the original system of equations.




3. The graph of the new equation goes through the intersection point of the original two lines. The sum of the original equations results in a graph that passes through the same point as the solution to the original linear system.

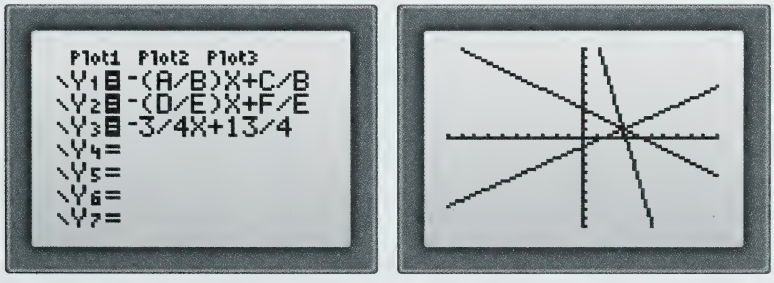
4. Subtract equation (2) from equation (1).

$$\begin{array}{rl} 5x+\ y=16 & (1) \\ \underline{2x-3y=\ 3} & (2) \\ 3x+4y=13 & \end{array}$$

Rearrange the equation to isolate $y$.

$$\begin{aligned} 3x+4y&=13 \\ 4y&=-3x+13 \\ y&=-\frac{3}{4}x+\frac{13}{4} \end{aligned}$$

5. Graph the equation by entering it into the equation editor. Leave $Y_1$ and $Y_2$ as filled in by the LINES program based on the original system of equations.




6. The graph of the new equation goes through the intersection point of the original two lines. The graph of the difference of the original equations passes through the same point as the solution to the original linear system.

7. Adding or subtracting the equations of a linear system results in a graph that passes through the same point as the solution to the original system.

**9. Textbook exercise 7 of "Exercises: Checking Your Skills," p. 148**

**7. a.** You would have to multiply equation (1) by 3 to make the coefficients of $x$ the same.

**b.** $6x+9y=15$ (3)

**c.** $6x+9y=15$ (3)

$6x+7y=1$ (2)

$2y=14$ ← Subtract equation (2) from equation (3).

$y=7$ ← Divide both sides by 2.

**d.** To find the value of $x$ at the point of intersection, substitute the value of $y$ into either of the original equations. Then solve the resulting equation for $x$.

**e.** Substitute 7 for $y$ into equation (1) to find the value of $x$.

$$\begin{aligned} 2x+3y&=5\\ 2x+3(7)&=5\\ 2x+21&=5\\ 2x&=-16\\ x&=-8 \end{aligned}$$

**10. Textbook exercise "Communicating the Ideas," p. 148**

Let a linear system of equations be represented by the following:

$Ax+By=C$ (1)

$Dx+Ey=F$ (2)

If $A:D=B:E\neq C:F$, the system has no solution.

If $A:D=B:E=C:F$, the system has an infinite number of solutions.

If $A:D\neq B:E$, the system has exactly one solution.

The explanation may be in paragraph form similar to the following:

> Given a linear system of equations, $Ax+By=C$ and $Dx+Ey=F$, suppose the ratio of $A$ to $D$ is equal to the ratio of $B$ to $E$. Then if the ratio of $C$ to $F$ is not equal to the ratio of $A$ to $D$, the linear system has no solution. If the ratio of $C$ to $F$ is equal to the ratio of $A$ to $D$, the linear system has an infinite number of solutions. If the ratio of $A$ to $D$ is not equal to the ratio of $B$ to $E$, the linear system has exactly one solution.

# Activity 5: Solving Linear Systems by Elimination

**1. Textbook exercise 1 of "Discussing the Ideas," p. 153**

**1.**
$$\begin{aligned} x+y&=10 && (1)\\ x-y&=6 && (2)\\ \hline 2y&=4 && (1)-(2)\\ y&=2 \end{aligned}$$

Substitute 2 for $y$ into equation (1).

$$\begin{aligned} x+y&=10\\ x+2&=10\\ x&=8 \end{aligned}$$

Subtracting the equations (eliminating $x$) leads to the same values for $x$ and $y$.

**2. Textbook exercises 1.b., 3, and 4 of "Exercises: Checking Your Skills," pp. 153 and 154**

**1. b.**
$$\begin{aligned} 3x+5y&=12 && (1)\\ 7x+5y&=8 && (2)\\ \hline -4x&=4 && (1)-(2)\\ x&=-1 && \leftarrow \text{Divide both sides by } -4. \end{aligned}$$

Substitute $-1$ for $x$ into equation (1).

$$\begin{aligned} 3x+5y&=12\\ 3(-1)+5y&=12\\ -3+5y&=12\\ 5y&=15\\ y&=3 \end{aligned}$$

Check if $x=-1$ and $y=3$ is a solution.

| LS | RS |
|---|---|
| $3x+5y$ | 12 |
| $=3(-1)+5(3)$ | |
| $=-3+15$ | |
| $=12$ | |

LS = RS

| LS | RS |
|---|---|
| $7x+5y$ | 8 |
| $=7(-1)+5(3)$ | |
| $=-7+15$ | |
| $=8$ | |

LS = RS

The ordered pair $(-1, 3)$ is a solution.

## Activity 5 (continued)

**3.**

$$\begin{array}{rll} s+w= & 800 & (1) \\ s-w= & 730 & (2) \\ \hline 2s & =1530 & (1)+(2) \\ s & =765 & \leftarrow \text{Divide both sides by 2.} \end{array}$$

Substitute 765 for $s$ into equation (1).

$$\begin{aligned} s+w &= 800 \\ 765+w &= 800 \\ w &= 35 \end{aligned}$$

Check if $s=765$ and $w=35$ is a solution.

| LS | RS |
|---|---|
| $s+w$ | 800 |
| $=765+35$ | |
| $=800$ | |

LS = RS

| LS | RS |
|---|---|
| $s-w$ | 730 |
| $=765-35$ | |
| $=730$ | |

LS = RS

The wind speed is 35 km/h, and the speed of the airplane in still air is 765 km/h.

**4.**

$$\begin{array}{rll} w+ & c= & 300 \quad (1) \\ w+2c= & & 500 \quad (2) \\ \hline -c & =-200 & (1)-(2) \\ c & =200 & \end{array}$$

Here, $w$ is the number of wheelbarrows and $c$ is the number of carts.

Substitute 200 for $c$ into equation (1).

$$\begin{aligned} w+c &= 300 \\ w+200 &= 300 \\ w &= 100 \end{aligned}$$

Check if $w = 100$ and $c = 200$ is a solution.

| LS | RS |
|---|---|
| $w + c$ | 300 |
| $= 100 + 200$ | |
| $= 300$ | |

LS = RS

| LS | RS |
|---|---|
| $w + 2c$ | 500 |
| $= 100 + 2(200)$ | |
| $= 100 + 400$ | |
| $= 500$ | |

LS = RS

Therefore, 100 wheelbarrows and 200 carts can be made.

**3. Textbook exercises 2 and 3 of "Discussing the Ideas," p. 153**

**2.** $x + y = 8000$ (1)
$50x + 30y = 250\,000$ (2)

**a.** To eliminate $x$ by adding, you need a coefficient of $-50$ for $x$ in equation (1). Therefore, you will have to multiply each term in equation (1) by $-50$.

**b.** To eliminate $y$ by subtracting, you need a coefficient of 30 for $y$ in equation (1). Therefore, you will need to multiply each term in equation (1) by 30.

**c.** To eliminate $y$ by adding, you need a coefficient of $-30$ for $y$ in equation (1). Therefore, you will need to multiply each term in equation (1) by $-30$.

**3.** Equation (1) was chosen because it is simpler to solve for $x$. Substituting 7500 for $y$ into equation (2) would give the same answer.

Since $y = 7500$ is part of the solution of the equation, both equations have the same $x$-value for this $y$-value.

**4. Textbook exercises 2.b., 2.c., and 5 of "Exercises: Checking Your Skills," pp. 153 and 154**

**2. b.** $5x + 2y = 5$ (1)
$3x - 4y = -23$ (2)

$10x + 4y = 10$ (3): 2 × (1)
$3x - 4y = -23$ (2)
$13x = -13$ (3) + (2)
$x = -1$

## Activity 5 (continued)

Substitute $-1$ for $x$ into equation (2).

$$\begin{aligned} 3x-4y&=-23\\ 3(-1)-4y&=-23\\ -3-4y&=-23\\ -4y&=-20\\ y&=5 \end{aligned}$$

Check if $x=-1$ and $y=5$ is a solution.

| LS | RS |
|---|---|
| $5x+2y$ | 5 |
| $=5(-1)+2(5)$ | |
| $=-5+10$ | |
| $=5$ | |

LS = RS

| LS | RS |
|---|---|
| $3x-4y$ | $-23$ |
| $=3(-1)-4(5)$ | |
| $=-3-20$ | |
| $=-23$ | |

LS = RS

The solution is $(-1, 5)$.

**c.** $6x-5y=-2$ (1)
$2x+3y=18$ (2)

$$\begin{array}{rll} 6x-5y=&-2 & (1)\\ 6x+9y=&54 & (3):3\times(2)\\ \hline -14y=&-56 & (1)-(3)\\ y=&4 & \end{array}$$

Substitute 4 for $y$ into equation (2).

$$\begin{aligned} 2x+3y&=18\\ 2x+3(4)&=18\\ 2x+12&=18\\ 2x&=6\\ x&=3 \end{aligned}$$

Check if $x = 3$ and $y = 4$ is a solution.

| LS | RS |
|---|---|
| $6x - 5y$ | $-2$ |
| $= 6(3) - 5(4)$ | |
| $= 18 - 20$ | |
| $= -2$ | |

LS = RS

| LS | RS |
|---|---|
| $2x + 3y$ | 18 |
| $= 2(3) + 3(4)$ | |
| $= 6 + 12$ | |
| $= 18$ | |

LS = RS

The solution is $(3, 4)$.

**5.** $c + h = 600$ ①

$0.11c + 0.08h = 62$ ②

$0.11c + 0.11h = 66$ ③ : $0.11 \times$ ①

$0.11c + 0.08h = 62$ ②

$0.03h = 4$ ③ − ②

$h = 133.\overline{3}$

$\doteq 133$ ← Round your answer.

Substitute 133.3 for $h$ into equation ①.

$c + h = 600$

$c + 133.3 \doteq 600$

$c \doteq 467$ ← Round your answer.

When using rounded values, use one extra decimal place when calculating the value of the other variable.

Check if $c \doteq 467$ and $h \doteq 133$ is a solution.

| LS | RS |
|---|---|
| $c + h$ | 600 |
| $\doteq 467 + 133$ | |
| $\doteq 600$ | |

LS = RS

| LS | RS |
|---|---|
| $0.11(467) + 0.08(133)$ | 62 |
| $\doteq 51.37 + 10.64$ | |
| $\doteq 62$ | |

LS = RS

The distance driven in the city was about 467 km, and the distance driven on the highway was about 133 km.

## Activity 5 (continued)

**5. Textbook exercises 4, 5, and 6 of "Discussing the Ideas," p. 153**

**4.** In equation ①, there is $5s$; in equation ②, there is $12s$. Make both coefficients 120, since it is a multiple of both coefficients. Then you would need to multiply equation ① by $\frac{120}{5} = 24$ and equation ② by $\frac{120}{12} = 10$.

Another common multiple of the coefficients of $s$ is 180. You could make 180 the new coefficients of $s$. Here, you would need to multiply equation ① by $\frac{180}{5} = 36$ and equation ② by $\frac{180}{12} = 15$.

**5.** Answers may vary. If you use the substitution method, you end up using a cumbersome expression and a complicated equation. Solving for $d$ or $s$ leads to a cumbersome expression, like $d = \frac{1700 - 5s}{8}$.

When you substitute the equivalent expression for $d$ or $s$, you end up with a complicated equation. For example, substituting $\frac{1700-5s}{8}$ for $d$ into equation ② yields $30\left(\frac{1700-5s}{8}\right) + 12s = 5700$.

**6.** You add equations when the coefficients of one of the varibles are opposites (additive inverses). You subtract equations when the coefficients of one of the variables have the same sign.

**6.** $2x + 5y = 39$ ①
$3x - 4y = -22$ ②

Aim to make 6 the coefficient of $x$ in both equations because 6 is the lowest common multiple of 2 and 3.

Therefore, multiply equation ① by $\frac{6}{2} = 3$ and equation ② by $\frac{6}{3} = 2$.

$$\begin{aligned} 6x + 15y &= 117 \quad ③ : 3 \times ① \\ 6x - 8y &= -44 \quad ④ : 2 \times ② \\ \hline 23y &= 161 \quad ③ - ④ \\ y &= 7 \end{aligned}$$

Substitute 7 for $y$ into equation ①.

$$\begin{aligned} 2x + 5y &= 39 \\ 2x + 5(7) &= 39 \\ 2x + 35 &= 39 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Check if $x=2$ and $y=7$ is a solution.

| LS | RS |
|---|---|
| $2x+5y$ | 39 |
| $=2(2)+5(7)$ | |
| $=4+35$ | |
| $=39$ | |

LS = RS

| LS | RS |
|---|---|
| $3x-4y$ | $-22$ |
| $=3(2)-4(7)$ | |
| $=6-28$ | |
| $=-22$ | |

LS = RS

The solution is $(2, 7)$.

**7. Textbook exercises 6, 7, and 11 of "Exercises: Checking Your Skills," pp. 154 to 156**

**6. a.** $v+t=100$ (1)

**b.** $8v+12t=980$ (2)

**c.**
$$\begin{array}{rl} 8v+8t=800 & (3):8\times(1) \\ 8v+12t=980 & (2) \\ \hline -4t=-180 & (3)-(2) \\ t=45 & \end{array}$$

Substitute 45 for $t$ into equation (1).

$$\begin{aligned} v+t&=100 \\ v+45&=100 \\ v&=55 \end{aligned}$$

Check if $v=55$ and $t=45$ is a solution.

| LS | RS |
|---|---|
| $v+t$ | 100 |
| $=55+45$ | |
| $=100$ | |

LS = RS

| LS | RS |
|---|---|
| $8v+12t$ | 980 |
| $=8(55)+12(45)$ | |
| $=440+540$ | |
| $=980$ | |

LS = RS

Therefore, 55 plates of vegetarian lasagna and 45 plates of roast turkey can be served.

## Activity 5 (continued)

**7. a.** Let $x$ be the number of racing boards and $y$ be the number of free-style boards.

**b.**

| | Cost of Materials ($) | Cost of Labour ($) |
|---|---|---|
| Racing board | $120\,x$ | $180\,x$ |
| Free-style board | $100\,y$ | $150\,y$ |
| Total | 49 000 | 73 500 |

$$\therefore\ 180x + 150y = 73\,500 \quad (1)$$
$$120x + 100y = 49\,000 \quad (2)$$

**c.**
$$360x + 300y = 147\,000 \quad (3) : 2 \times (1)$$
$$\underline{360x + 300y = 147\,000} \quad (4) : 3 \times (2)$$
$$0 = 0 \quad (3) - (4)$$

Notice that subtraction yields an equation without the variables. The equations impose no restrictions on $x$ and $y$.

Any ordered pair that satisfies one equation of the system will satisfy the other. Therefore, the system has an infinite number of solutions. However, in this problem you only consider solutions in which $x$ and $y$ are counting numbers (representing whole boards). This still leaves many solutions.

> Looking back on equations (1) and (2) of the system you can see that the corresponding coefficients and the constant terms occur in the same ratio, 3:2. That means the graph of the system consists of two coincident lines.

Any point satisfying one equation is a solution of the system.

Equation (1)

$$y = -\frac{180}{150}x + \frac{73\,500}{150}$$
$$y = -\frac{6}{5}x + 490$$

Equation (2)

$$y = -\frac{120}{100}x + \frac{49\,000}{100}$$
$$y = -\frac{6}{5}x + 490$$

Try $x = 100$.

$$\begin{aligned} y &= -\frac{6}{5}x + 490 \\ &= -\frac{6}{5}(100) + 490 \\ &= -120 + 490 \\ &= 370 \end{aligned}$$

Some solutions are $(100, 370)$, $(200, 250)$, and $(150, 310)$. Therefore, some numbers of each type of board that could be produced are as follows:

- 100 racing boards and 370 free-style boards
- 200 racing boards and 250 free-style boards
- 150 racing boards and 310 free-style boards

There are many other solutions that fit the conditions.

**11. a.** Let $x$ be the weight (in pounds) of French roast in the blend and $y$ be the weight (in pounds) of Kenyan roast in the blend.

**b.**

| | Weight (pounds) | Cost ($/pound) |
|---|---|---|
| French roast | $x$ | $4.95x$ |
| Kenyan roast | $y$ | $3.75y$ |
| Total | 50 | 217.50 |

$\therefore\ x + y = 50$ (1)

$4.95x + 3.75y = 217.50$ (2)

To find the total cost of the blend, multiply the cost per pound by the number of pounds.

$4.35 \times 50 = 217.50$

**c.**

$4.95x + 4.95y = 247.50$ (3) : $4.95 \times$ (1)

$4.95x + 3.75y = 217.50$ (2)

$1.20y = 30.00$ (3) − (2)

$y = 25$

Substitute 25 for $y$ into equation (1).

$$\begin{aligned} x + y &= 50 \\ x + 25 &= 50 \\ x &= 25 \end{aligned}$$

## Activity 5 (continued)

Check if $x = 25$ and $y = 25$ is a solution of the system.

| LS | RS |
|---|---|
| $x + y$ | 50 |
| $= 25 + 25$ | |
| $= 50$ | |

LS = RS

| LS | RS |
|---|---|
| $4.95x + 3.75y$ | 217.50 |
| $= 4.95(25) + 3.75(25)$ | |
| $= 123.75 + 93.75$ | |
| $= 217.50$ | |

LS = RS

To make the house blend, 25 pounds of French roast coffee should be mixed with 25 pounds of Kenyan roast coffee.

**8. Textbook exercises 12 and 13 of "Exercises: Extending Your Thinking," p. 157**

**12.**

| | Volume (L) | Amount of Salt (L) |
|---|---|---|
| 10% solution | $x$ | $0.10x$ |
| 5% solution | $y$ | $0.05y$ |
| 8% solution | 4 | 0.32 ← 8% of 4 L |

**a.** $x + y = 4$ (1)

**b.** $0.10x + 0.05y = 0.32$ (2)

**c.**
$$\begin{aligned} 0.10x + 0.10y &= 0.40 && (3) : 0.10 \times (1) \\ 0.10x + 0.05y &= 0.32 && (2) \\ \hline 0.05y &= 0.08 && (3) - (2) \\ y &= 1.6 \end{aligned}$$

Substitute 1.6 for $y$ into equation (1).

$$\begin{aligned} x + y &= 4 \\ x + 1.6 &= 4 \\ x &= 2.4 \end{aligned}$$

The nurse will use 2.4 L of 10% saline solution and 1.6 L of 5% saline solution to prepare the 4 L of 8% saline solution.

**13. a.**

| | Mass of Alloy (g) | Mass of Silver (g) |
|---|---|---|
| 60% silver alloy | $x$ | $0.60x$ |
| 35% silver alloy | $y$ | $0.35y$ |
| 50% silver alloy | 500 | 250 ← 50% of 500 g |

$\therefore\ x+y=500$ ①

$0.60x+0.35y=250$ ②

**b.** $0.60x+0.60y=300$ ③ : $0.60\times$ ①

$0.60x+0.35y=250$ ②

$0.25y=50$ ③ − ②

$y=200$

Substitute 200 for $y$ into equation ①.

$x+y=500$

$x+200=500$

$x=300$

The jeweller used 300 g of 60% silver alloy and 200 g of 35% silver alloy to make the final alloy.

**9.** Answers will vary. Sample answers are given.

**a.**

First, write an equation so one variable can be easily isolated. (It should have 1 as a coefficient.)

Second, when choosing the coefficients for equation ②, make sure the corresponding coefficients do not form the same ratio.

$9x-y=14$ ①

$2x+3y=16$ ②

Third, determine the constant terms by substituting a solution into the equations. For example, let $x=2$ and $y=4$.

$9(2)-4=18-4$ and $2(2)+3(4)=4+12$

$=14$ $=16$

## Activity 5 (continued)

**b. Substitution Method**

Solve equation (1) for $y$.

$$\begin{aligned} 9x - y &= 14 \\ -y &= -9x + 14 \\ y &= 9x - 14 \end{aligned}$$

Substitute $9x - 14$ for $y$ into equation (2).

$$\begin{aligned} 2x + 3y &= 16 \\ 2x + 3(9x - 14) &= 16 \\ 2x + 27x - 42 &= 16 \\ 29x - 42 &= 16 \\ 29x &= 58 \\ x &= 2 \end{aligned}$$

Substitute 2 for $x$ into equation (1).

$$\begin{aligned} 9x - y &= 14 \\ 9(2) - y &= 14 \\ 18 - y &= 14 \\ -y &= -4 \\ y &= 4 \end{aligned}$$

The solution is $(2, 4)$.

**Elimination Method**

$$\begin{array}{rll} 27x - 3y = 42 & & (3) : 3 \times (1) \\ 2x + 3y = 16 & & (2) \\ \hline 29x = 58 & & (3) + (2) \\ x = 2 & & \end{array}$$

Substitute 2 for $x$ into equation (1).

$$\begin{aligned} 9x - y &= 14 \\ 9(2) - y &= 14 \\ 18 - y &= 14 \\ -y &= -4 \\ y &= 4 \end{aligned}$$

**c.** Answers may vary. A sample answer is given.

In this case, the elimination method is better for solving the system of equations. It is shorter and does not involve mutiplying terms across brackets.

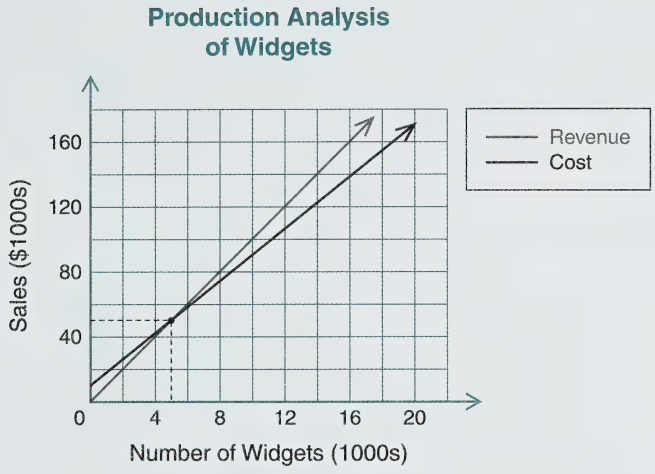
# Follow-up Activities

**Textbook exercises 1.a. to 1.h. of Part A of "What Should I Be Able To Do?," p. 161**

**1. a.** $C = 8n + 10\,000$

$R = 10n$

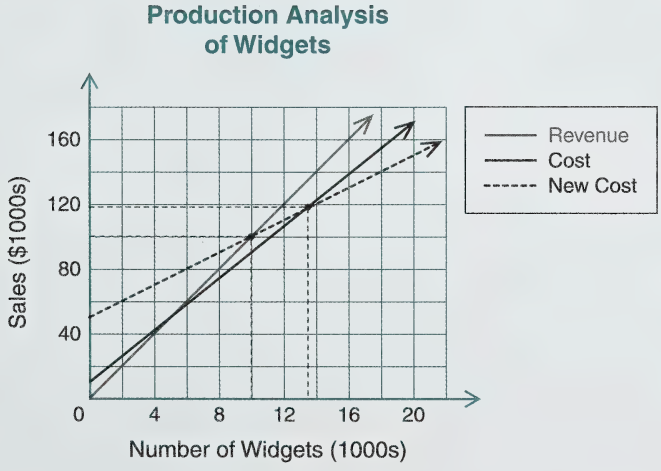
**b.**




For the business to break even, 5000 widgets must be sold. The production costs and revenue at this point is $50 000.

**c.** Let $C_N$ represent the new cost graph.

$\therefore\ C_N = 5n + 50\,000$



## Follow-up Activities (continued)

**d.** The new equipment becomes cost-effective when approximately 13 300 widgets are produced.

**e.** The new break-even point occurs when 10 000 widgets are produced.

**f.** Let $y$ represent the output for the two relations

$$\therefore\ y = 5n + 50\,000 \quad (1) \quad \leftarrow \text{New Cost}$$
$$y = 10n \quad (2) \quad \leftarrow \text{Revenue}$$

$$\begin{aligned} y &= 5n + 50\,000 && (1)\\ y &= 10n && (2)\\ \hline 0 &= -5n + 50\,000 && (1) - (2)\end{aligned}$$

$$5n = 50\,000$$
$$\frac{5n}{5} = \frac{50\,000}{5}$$
$$n = 10\,000$$

Substitute 10 000 for $n$ into equation (2).

$$\begin{aligned} y &= 10n\\ &= 10(10\,000)\\ &= 100\,000\end{aligned}$$

The new break-even point is $(10\,000, 100\,000)$.

**g.**
$$\begin{aligned} R &= 10n\\ &= 10(20\,000)\\ &= 200\,000\end{aligned}$$

$$\begin{aligned} C &= 8n + 10\,000\\ &= 8(20\,000) + 10\,000\\ &= 160\,000 + 10\,000\\ &= 170\,000\end{aligned}$$

$$\begin{aligned} C_N &= 5n + 50\,000\\ &= 5(20\,000) + 50\,000\\ &= 100\,000 + 50\,000\\ &= 150\,000\end{aligned}$$

$$\begin{aligned} \text{Old Profit} &= R - C\\ &= 200\,000 - 170\,000\\ &= 30\,000\end{aligned}$$

$$\begin{aligned} \text{New Profit} &= R - C_N\\ &= 200\,000 - 150\,000\\ &= 50\,000\end{aligned}$$

Once the new equipment is installed, $\$50\,000 - \$30\,000 = \$20\,000$ more profit can be made when 20 000 widgets are produced.

**h.** $R = 10n$
$= 10(15\,000)$
$= 150\,000$

$C = 8n + 10\,000$
$= 8(15\,000) + 10\,000$
$= 120\,000 + 10\,000$
$= 130\,000$

$C_N = 5n + 50\,000$
$= 5(15\,000) + 50\,000$
$= 75\,000 + 50\,000$
$= 125\,000$

Old Profit $= R - C$
$= 150\,000 - 130\,000$
$= 20\,000$

New Profit $= R - C_N$
$= 150\,000 - 125\,000$
$= 25\,000$

The company should purchase the new equipment if they sell an average of 15 000 units each month. The new equipment will increase profits by $5000.

## Extra Help

$5x + 6y = 13$ (1)
$7x + 4y = 5$ (2)

Notice that it is easier to find the LCM of the coefficients of $y$.

$\therefore\ 10x + 12y = 26$ (3) : $2 \times$ (1)
$21x + 12y = 15$ (4) : $3 \times$ (2)
$-11x \quad = 11$ (3) $-$ (4)

$2 \times 6y = 12y$
$3 \times 4y = 12y$

$\frac{-11x}{-11} = \frac{11}{-11}$ ← Divide both sides by $-11$ to isolate $x$.

$x = -1$

Substitute $-1$ for $x$ into equation (2).

$7x + 4y = 5$
$7(-1) + 4y = 5$
$-7 + 4y = 5$
$-7 + 4y + 7 = 5 + 7$ ← Add 7 to both sides to isolate $y$-term.
$4y = 12$
$\frac{4y}{4} = \frac{12}{4}$ ← Divide both sides by 4 to isolate $y$.
$y = 3$

## Follow-up Activities (continued)

Check if $x=-1$ and $y=3$ is a solution of the system.

| LS | RS |
|---|---|
| $5x+6y$ | 13 |
| $=5(-1)+6(3)$ | |
| $=-5+18$ | |
| $=13$ | |

LS = RS

| LS | RS |
|---|---|
| $7x+4y$ | 5 |
| $=7(-1)+4(3)$ | |
| $=-7+12$ | |
| $=5$ | |

LS = RS

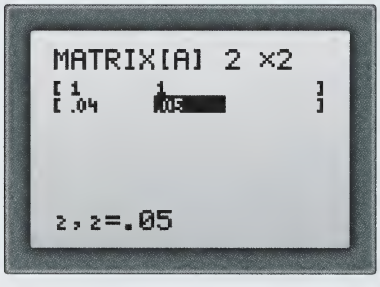
The solution is $(-1, 3)$.

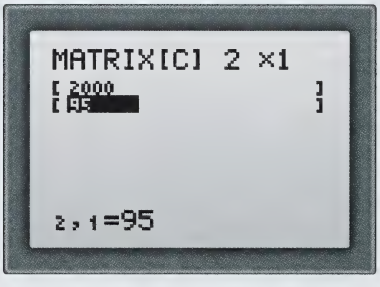
### Enrichment

$x+y=2000$ (1)

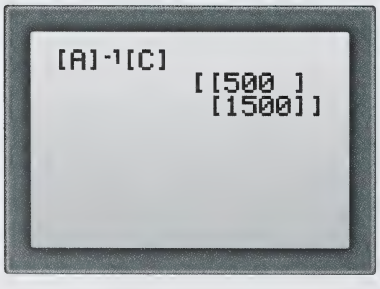
$0.04x+0.05y=95$ (2)

Enter matrix $A$ and matric $C$.




Determine the value of $A^{-1}C$.




Therefore, $x=500$ and $y=1500$.

Check if $(500, 1500)$ is a solution of the system.

| LS | RS |
|---|---|
| $x+y$ | 2000 |
| $=(500)+(1500)$ | |
| $=2000$ | |

LS = RS

| LS | RS |
|---|---|
| $0.04x+0.05y$ | 95 |
| $=0.04(500)+0.05(1500)$ | |
| $=20+75$ | |
| $=95$ | |

LS = RS

The solution is $(500, 1500)$.

## Module Project: Skydiving

### Completing the Project

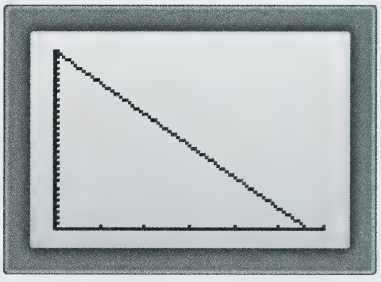
1. A large snowflake falls very slowly because of the great air resistance. The air resistance opposes the downward pull of gravity on the falling snowflake.

2. **Textbook exercises 1 and 2 of "Deploying a Parachute," pp. 140 and 141**

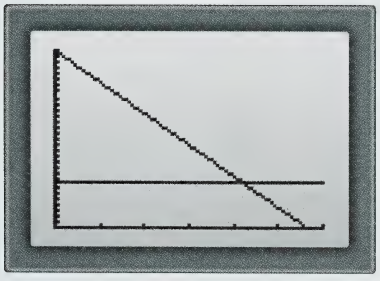
   1. **a.** Let $y$ represent the distance (in metres) above the ground and $x$ represent the elapsed time (in seconds).

      $\therefore\ y=-53x+3000$

      Your graph should be similar to the one on the right.

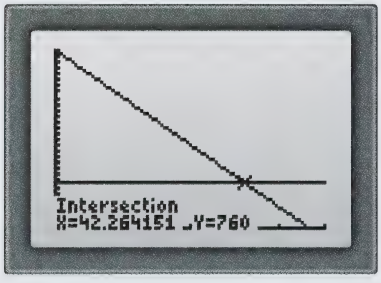
      

      **b.**

      The line $y=760$ represents the minimum height at which it is safe to deploy a parachute.

**c.** Use the Intersect feature from the CALCULATE menu to determine the point of intersection.




The intersection point is about $(42.3, 760)$. This point shows that the skydiver reaches a height of 760 m after approximately 42.3 s.

**d.** The skydiver has approximately 42.3 s of free fall time before the parachute must be deployed.

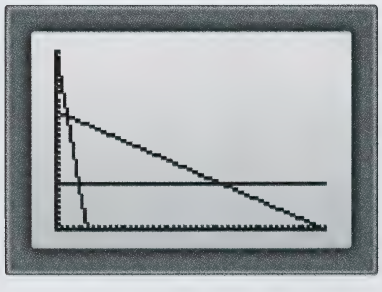
**2. a.** The maximum time of descent for a parachutist would occur if the parachute opened immediately after exiting the plane. The equation $y = -4x + b$ describes the height of the parachutist above ground, where $x$ is the time after exiting the plane, and $b$ is the height of the plane when the parachutist jumps.

Therefore, $y = -4x + b$ is the equation to determine the height above the ground at any time, $x$.

Suppose $b = 2000$.

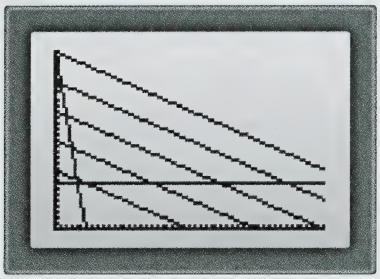
$\therefore\ y = -4x + 2000$

The graph (along with the previous ones) is as follows:



Reset Xmax to 500. **Note:** The graph in 1.c. is now the steeper graph in this graphing calculator window.

**b.** Answers will vary, since many values of $b$ are appropriate.



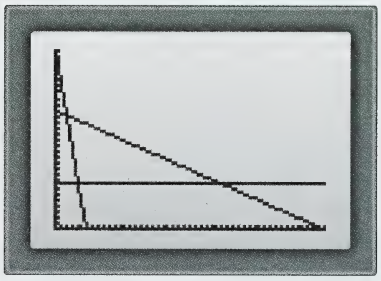
The graphs of the rate of descent with an open parachute given in the display are as follows:

- $y = -4x + 3000$
- $y = -4x + 2500$
- $y = -4x + 2000$
- $y = -4x + 1500$
- $y = -4x + 1000$

**Note:** $y = -53x + 3000$ is the equation of the line representing free fall. The other equations are for lines representing a descent with an open parachute from its intersection with the free fall line.

**c.** Answers will vary. A sample answer is given based on $b = 1500$.

On the graph, the intersection point of $y = -4x + 1500$ and $y = -53x + 3000$ seems high enough and early enough with respect to the last point for safe deployment.

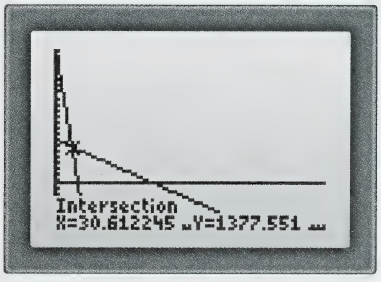


The points at which the parachute is to be deployed corresponds to the solution of this system:

$$y = -53x + 3000 \quad (1)$$
$$y = -4x + 1500 \quad (2)$$

Use the Intersect feature from the CALCULATE menu to find the solution.

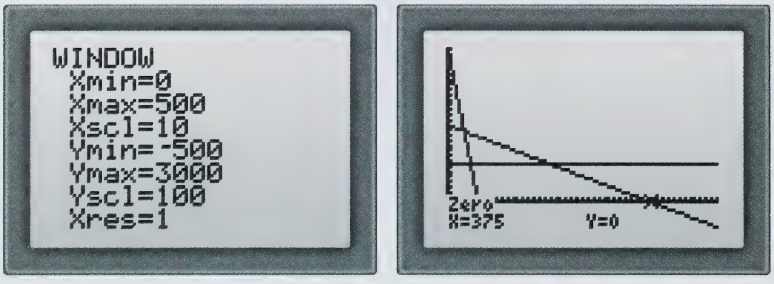
The solution is approximately $(30.6, 1378)$. Note that this is more than 10 s before the latest possible time for safe deployment as the value of $b$ that was chosen was at a much higher altitude than 760 m. The skydiver will fall at terminal velocity for almost 31 s.



**d.** The skydiver deployed the parachute at a height of 1378 m (where $b = 1500$).

**e.** Determine the $x$-intercept of $y = -4x + 1500$.

Using the window setting given, use the Zero feature from the CALCULATE menu. Select the equation, $y = -4x + 1500$, using ▼ or ▲ keys.




The $x$-intercept is $x = 375$.

The time elapsed for the complete jump is 375 s.

The parachute is opened 30.6 s into the jump. Therefore, it takes $375 - 30.6 \doteq 344$ s to reach the ground after the parachute is opened.

**3. Skydiver in Dive Position**

**Stage 1** Height after 2 s,

$$h = -4.9t^2 + 3600$$
$$= -4.9(2)^2 + 3600$$
$$= 3580.4 \text{ m}$$

Height after 8 s,

$$h = -4.9t^2 + 3600$$
$$= -4.9(8)^2 + 3600$$
$$= 3286.4 \text{ m}$$

These agree with the student's data.

**Stage 2** Height after 18.66 s,

$t = 18.66 - 8.66$

$= 10$ s

$h = -85t + 3232.2$

$= -85(10) + 3232.2$

$= 2382.2$ m

Height after 36.66 s,

$t = 36.66 - 8.66$

$= 28$ s

$h = -85t + 3232.2$

$= -85(28) + 3232.2$

$= 852.2$ m

These agree with the student's data.

**Stage 3** Height after 97 s,

$t = 97 - 37$

$= 60$ s

$h = -4t + 823.3$

$= -4(60) + 823.3$

$= 583.3$ m

Height after 217 s,

$t = 217 - 37$

$= 180$ s

$h = -4t + 823.3$

$= -4(180) + 823.3$

$= 103.3$ m

These agree with the student's data.

**Skydiver in Spread Stable Position**

**Stage 1** Height after 5 s,

Distance fallen $= 111.5$ m

$h = 3600 - 111.5$

$= 3488.5$ m

Height after 12 s,

Distance fallen $= 452.5$ m

$h = 3600 - 452.5$

$= 3147.5$ m

from table

These agree with the student's data.

**Stage 2**

Height after 30 s,

$$t = 30 - 12$$
$$= 18 \text{ s}$$

$$h = -53t + 3147.5$$
$$= -53(18) + 3147.5$$
$$= 2193.5 \text{ m}$$

Height after 54 s,

$$t = 54 - 12$$
$$= 42 \text{ s}$$

$$h = -53t + 3147.5$$
$$= -53(42) + 3147.5$$
$$= 921.5 \text{ m}$$

These agree with the student's data.

**Stage 3**

Height after 197 s,

$$t = 197 - 57$$
$$= 140 \text{ s}$$

$$h = -4t + 762.5$$
$$= -4(140) + 762.5$$
$$= 202.5 \text{ m}$$

Height after 247.625 s,

$$t = 247.625 - 57$$
$$= 190.625 \text{ s}$$

$$h = -4t + 762.5$$
$$= -4(190.625) + 762.5$$
$$= 0 \text{ m}$$

These agree with the student's data.

**4.** The data for the three stages are identified correctly except for the division between Stage 2 and Stage 3 of the Dive data. Stage 3 actually starts on the row where time is 36.66 s.

**5.** The portions of the graph that correspond to Stages 2 and 3 are straight because speed is constant.

# LINES

The following shows all of the keystrokes involved in entering the LINES program into your TI-83 graphing calculator.

Begin by opening the program window by pressing PRGM. Using the arrow keys, select the NEW menu.




Press ENTER. Now, the calculator prompts you to name the program. Name the program "LINES."




Press ENTER. Now the calculator prompts you to enter the lines of the program.



## LINES (continued)

Enter the lines of the program (as given on page 422 of the textbook) by pressing the following:

Select the Y-Vars menu.

VARS ▶ 4 (4:On/Of…) 2 (2:FnOff) ENTER

Select the Y-Vars menu.

VARS ▶ 4 (4:On/Of…) 1 (1:FnOn) 1 , 2 ENTER

Select the I/O menu.

PRGM ▶ 2 (2:Prompt) ALPHA [ A ] , ALPHA [ B ] , ALPHA [ C ] , ALPHA [ D ] ,
ALPHA [ E ] , ALPHA [ F ] ENTER

Select the I/O menu.

PRGM ▶ 4 (4:DispGraph) 2nd [ QUIT ]

**Note:** The word *Quit* will not appear on the display.

To run the program, press PRGM and select "LINES" under the EXEC menu. **Remember:** Before using the LINES program, you must enter $Y_1 = -(A/B)X + C/B$ and $Y_2 = -(D/E)X + F/E$ into the equation editor.

## IMAGE CREDITS

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Tom Buckler

**Page**

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